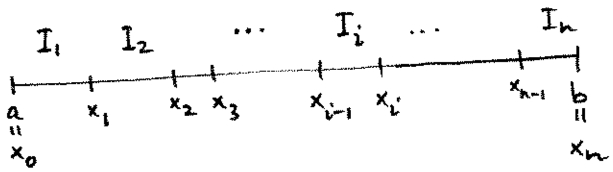
Math 1210-3
Friday March 28

§ 4.1-4.2 The definite integral

To warm up for the definition of $\int_a^b f(x)\,dx$, do pages 3-4 Wednesday notes.

After that, here's the precise definition, which we saw very briefly in polynomial calculus (jan15.pdf)

- Let $f(x)$ have domain containing the closed interval $[a,b]$.
- Let $P$ be a partition of $[a,b]$ into $n$ subintervals (where $n$ is some positive integer) $I_1, I_2, \ldots I_n$




$$I_i = [x_{i-1}, x_i]$$
$$\Delta x_i = \text{width} = x_i - x_{i-1}$$

In each subinterval pick a sample point $\bar{x}_i$, $x_{i-1} \le \bar{x}_i \le x_i$

Then, for this partition and choice of sample points, the <u>Riemann sum</u> $R_P$ for $f$ is

- $$R_P = \sum_{i=1}^{n} f(\bar{x}_i)\,\Delta x_i$$

- If $P$ is a partition as above, the "norm of $P$" $\|P\|$ is the maximum $\Delta x_i$, i.e. the maximum width of the subintervals

- If $\lim_{\|P\| \to 0} \sum_{i=1}^{n} f(\bar{x}_i)\,\Delta x_i$ exists, we call the limit the (definite) integral of $f(x)$, from $x=a$ to $x=b$, and write the limit as $\int_a^b f(x)\,dx$

- <u>Theorem</u>: If $f$ is continuous on $[a,b]$, then

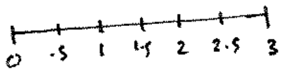
$$\lim_{\|P\| \to 0} R_P = \int_a^b f(x)\,dx \text{ exists.}$$

<u>One</u> interpretation of this integral is as the signed area between the graph of $f$ and the $x$-axis

## Exercise 1   We illustrate all the ideas on page 1!

Let $f(x) = x+2$ ,   $0 \le x \le 3$

Partition $[0,3]$ into 6 subintervals of length $3/6 = .5$




1a) $\|P\| =$

1b) Using right-hand endpoints as sample points yields

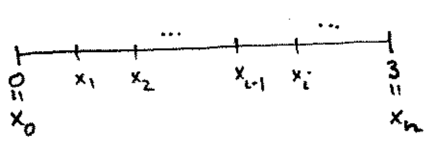
$$R_p = \sum_{i=1}^{6} f(\bar{x}_i)\Delta x_i =$$

1c) Using left-hand endpoints as sample points yields

$$R_p = \sum_{i=1}^{6} f(\bar{x}_i)\Delta x_i =$$

1d) Partition $[0,3]$ into "n" subintervals of equal length $\Delta x$.

Then   $\Delta x =$

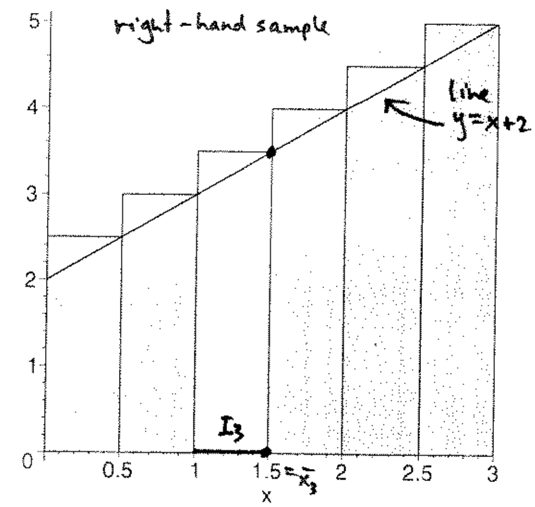


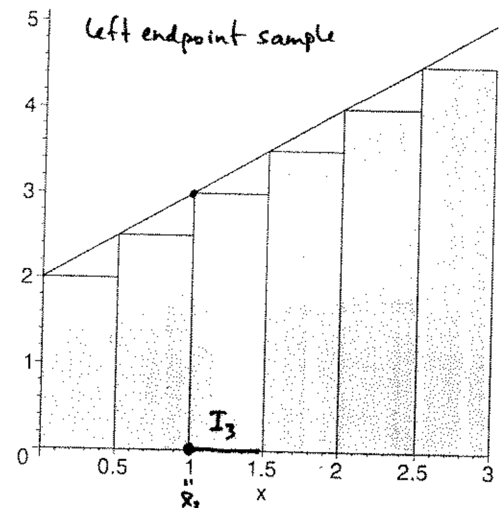

$$I_i = [x_{i-1}, x_i] = [\qquad , \qquad ]$$

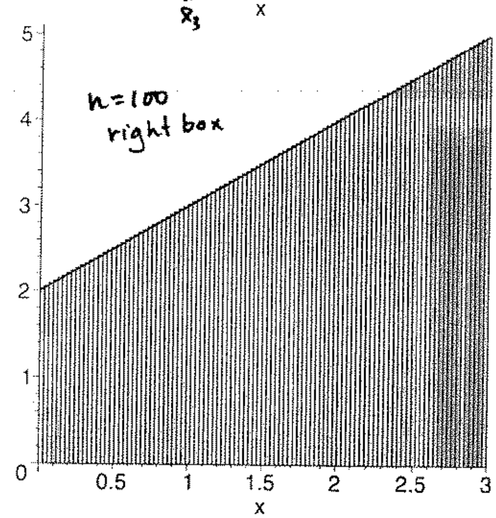
1e) Using right-hand endpoints as sample points, i.e. $\bar{x}_i = x_i$, yields

$$R_p = \sum_{i=1}^{n} f(\bar{x}_i)\Delta x = \sum_{i=1}^{n}$$

Riemann sums as areas




$$\frac{3}{100}\left(\sum_{i=1}^{100}\left(\frac{3\,i}{100}+2\right)\right)$$

$$= 10.54500000$$

1f) If we let $n \to \infty$, then $\|P\| = 3/n \to 0$,

and $$\int_0^3 x+2\,dx = \lim_{n\to\infty} R_P = \lim_{n\to\infty} \sum_{i=1}^{n} f(\bar{x}_i)\,\Delta x_i = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3i}{n}+2\right)\left(\frac{3}{n}\right)$$

Evaluate this limit using Arithmetic and the first magic formula at the bottom of this page. Your answer should be the exact trapezoid area!

Summation notation:

$$\sum_{i=1}^{n} b_i = b_1 + b_2 + \dots + b_n$$

Arithmetic:

$$\sum_{i=1}^{n} b_i + c_i = \sum_{i=1}^{n} b_i + \sum_{i=1}^{n} c_i$$

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} k\,b_i = k \sum_{i=1}^{n} b_i$$

Magic formulas: (page 218)

$$\sum_{i=1}^{n} i \quad (= 1+2+3+\dots+n) = \frac{1}{2}n(n+1)$$

$$\sum_{i=1}^{n} i^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$\sum_{i=1}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

these can be explained!

etc.?