Math 1210-3
Wednesday March 26

Friday Quiz §3.5:
Graphing functions with Calculus
(this section uses material from 3.1-3.3 too.)

§3.9 Introduction to differential equations

and, time permitting, return to the "definite integral", which we discussed very briefly in polynomial calculus – this is the content of Chapter 4.

pages 2.5-3 on Tuesday's notes.

Then, notice those examples were just antidifferentiation in disguise, because if

$$\frac{dy}{dx} = f(x)$$

is the DE, then the sol'ns are just

$$y = \int f(x)\,dx$$

There are slightly harder DE's which we can also solve, using the magic of differentials to do the chain rule backwards. They are separable differential equations :

$$\frac{dy}{dx} = \frac{f(x)}{g(y)}$$

Mathematically rigorous way to get sol'ns

$$g(y)\frac{dy}{dx} = f(x)$$

$$\int g(y)\frac{dy}{dx} = \int f(x)\,dx$$

$$\boxed{G(y(x)) = F(x) + C}$$

where $D_y G(y) = g(y)$
$D_x F(x) = f(x)$

an implicit eq'n for y as a function of x

differential magic, to get correct answer

$$\frac{dy}{dx} = \frac{f(x)}{g(y)}$$

$$g(y)\,dy = f(x)\,dx$$

$$\int g(y)\,dy = \int f(x)\,dx$$

$$G(y) = F(x) + C$$

same ans!

Exercise 1 (Example 2 p.205)

Solve

$$\begin{cases} \dfrac{dy}{dx} = \dfrac{x+3x^2}{y^2} \\ y(0) = 6 \end{cases}$$

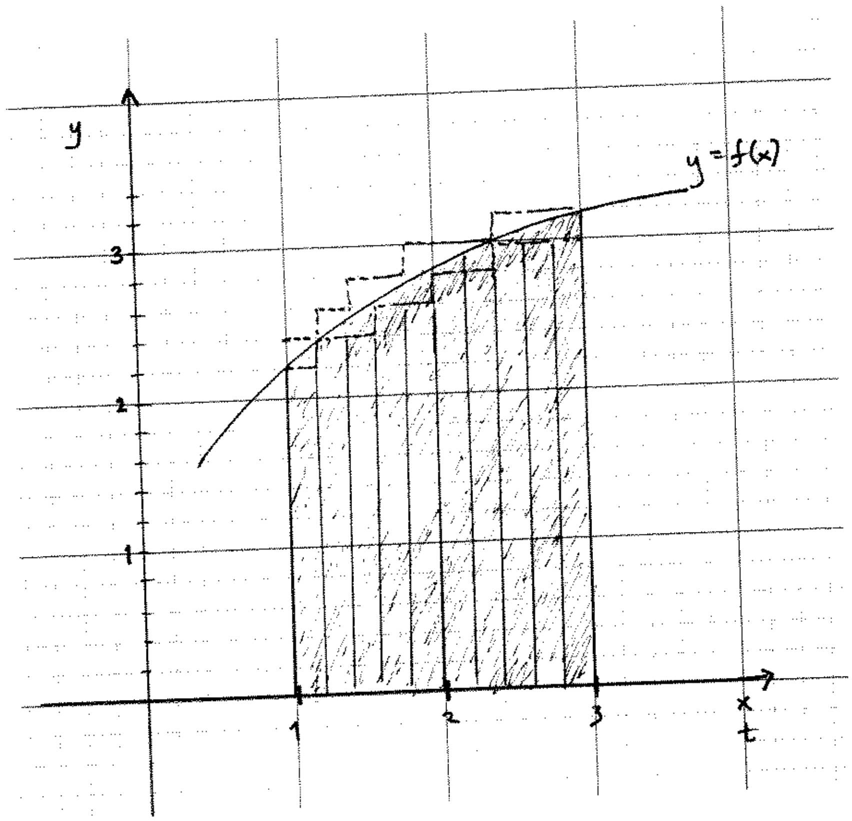
Exercise 2 Solve

$$\begin{cases} \dfrac{du}{dt} = u^2 \cos(2t) \\ u(0) = 1 \end{cases}$$

Chapter 4: The Definite Integral

§4.1 Introduction to Area, and to the definite integral ... we talked about this very briefly during polynomial calculus.

Consider the shaded region below, between the graph $y = f(x)$ and the x-axis, and with $1 \le x \le 3$.




Exercise 3: Estimate the area above and below, using the indicated tall rectangles of with $\Delta x = .2$. There are 10 of them.

Notation: Let $a_1, \ldots a_{10}$ be the smaller areas, $(a_1 = (.2)(2.2)$, for example$)$
" $A_1 .. A_{10}$ " larger " $(A_1 = (.2)(2.4)$, for example$)$

$$a_1 + a_2 + \cdots + a_{10} = \sum_{i=1}^{10} a_i =$$

$$A_1 + A_2 + \cdots + A_{10} = \sum_{i=1}^{10} A_i =$$

"The sum of $A_i$, as $i$ ranges from 1 to 10"

Exercise 4 Same picture as Exercise 3

Except now, the horizonal axis is $t$ (minutes)

and the vertical axis is $y$ (feet)/minute.

any $y = f(t)$ is the velocity of a turtle walking along a path.

What do your Exercise 1 estimates mean now?

Exercise 5 Same picture as Exercise 3

Same question as Exercise 4.

Except now, horizontal scale is hours

vertical scale is thousands of gallons/hour

and $y = f(t)$ is flow rate of water past a measuring station on a stream

Moral : the approximation in Exercise 3 (84.5) has many different interpretations

The limit of this approximation procedure as the x-interval widths $\to 0$ is called the definite integral, and in this case since $1 \le x \le 3$ we would write $\int_1^3 f(x)dx$ (or $\int_1^3 f(t)dt$ )

From our work in Exercise 3 we know $5.36 \le \int_1^3 f(x)dx \le 5.84$.

26.8
29.2
5.36
5.84