Math 1210-3
Tuesday March 25

§3.8–3.9 ← intro to differential equations
↑
antiderivatives and indefinite integrals
↑
finish pages 3-4 Monday notes.

On page 4 we show how using substitutions and differentials helps us in figuring out how to use the chain rule backwards in antidifferentiation problems. Here's why it works:

If $F'(u) = f(u)$

then
$$\int f(g(x))\,g'(x)\,dx = F(g(x)) + C$$

because $D_x F(g(x)) = F'(g(x)) \cdot g'(x)$ by the chain rule.
$= f(g(x))\,g'(x)$.

This is what you'll get if you use differential magic:

$$\int f(\underbrace{g(x)})\,g'(x)\,dx$$

let $u = g(x)$
then $du = g'(x)\,dx$

↓ substitute

$$= \int f(u)\,du$$

$= F(u) + C$ where $F'(u) = f(u)$

↓ unsubstitute

$= F(g(x)) + C$ Correct answer, so magic is O.K. to use.!

If we want another substitution problem, let's try

Exercise 1 $\displaystyle\int \frac{36}{(3t+2)^3}\,dt$

If we want to try a "product rule backwards" problem, which you'll focus on in 1220 under the name "integration by parts"

try

Exercise 2 $\displaystyle\int x\cos x\,dx$

§3.9 Differential Equations

an equation involving an unknown function, e.g. $y=f(x)$, along with its derivatives. Further conditions on y may also be specified. Solving the differential equation means finding the solution(s) $y=f(x)$.

Differential equations arise naturally as descriptions of how nature works (physics, biology, chemistry, engineering, etc.), and many of you will take Math 2250 which is mostly just about this topic.

Math 1210 example:

Exercise 3 Solve the differential equation $\frac{dy}{dx}=x$ for $y=f(x)$,

with the "initial condition $y(0)=1$.

Exercise 4 The acceleration of an object along a coordinate line is given by $a(t) = 36(3t+2)^{-3}$ meters$/s^2$.
If the velocity at $t=0$ is $5/2$ m/s, what is $v(t)$?

4a) What initial value problem (i.e. differential equation & initial condition) does $v(t)$ solve?

4b) Find $v(t)$, i.e. solve the I.V.P. in 4a).

4c) How fast is object traveling at $t=2$ sec?

4d) How far did object go between $t=0$ and $t=2$ sec?