

Math 1210-2
Tuesday Jan 8

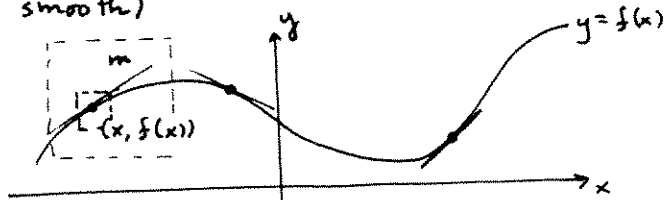
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P.2 Slope of a curve

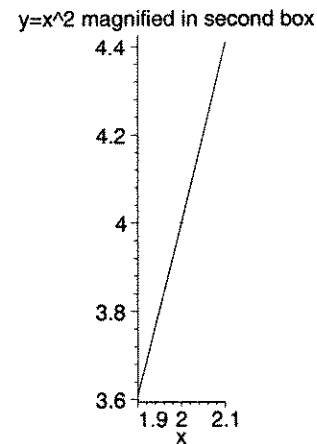
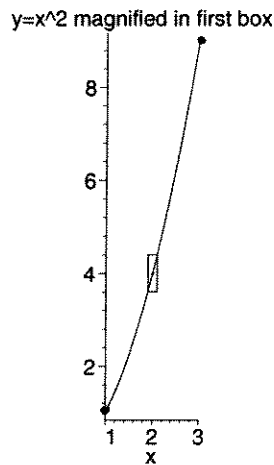
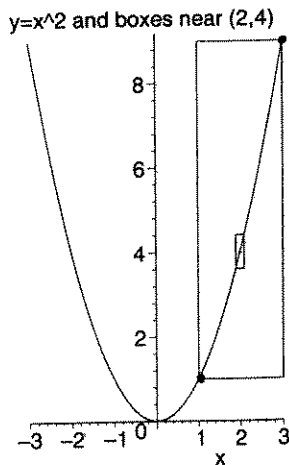
A graph $y = f(x)$ has a "slope" at each point $(x, f(x))$ on the graph (if the graph is smooth)

One way to visualize this slope is as the slope of the line "tangent" to the graph at $(x, f(x))$



Another way is to magnify the graph so much that it looks like a straight line (almost), and use that line's slope as an approximation.

Example 1 Estimate the slope of the parabola $y = x^2$ at the point $(2, 4)$, by magnification



How can we calculate an exact value of the slope?

We are still focused on the graph $y = x^2$, and the point $(2, 4)$

To save work we shall use $(2, 4)$ as base point,

and pick nearby points (with x near 2) as second points
with which to compute $\frac{\text{rise}}{\text{run}}$

Exercise 2

$$(2, 4) \text{ and } (3, 9) \quad \frac{\text{rise}}{\text{run}} = m_{\text{sec}} =$$

$$(2, 4) \text{ and } (1, 1) \quad \frac{\text{rise}}{\text{run}} = m_{\text{sec}} =$$

$$(2, 4) \text{ and } (2.1, (2.1)^2) \quad m_{\text{sec}} =$$

$$(2, 4) \text{ and } (2.01, (2.01)^2) \quad m_{\text{sec}} =$$

geometrically, we are computing the slopes of
"secant lines" to the graph $y = x^2$, thru $(2, 4)$:
and other nearby points on the graph

Now for the exact value:

base point $(2, 4)$

nearby points $(2+h, (2+h)^2)$

$$m_{\text{sec}} = \frac{(2+h)^2 - 4}{h} = \frac{4 + 4h + h^2 - 4}{h} = 4 + h$$

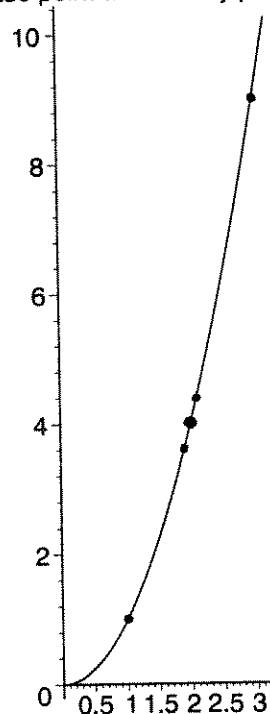
as h approaches zero, m_{sec} approaches 4

We write

$$\lim_{h \rightarrow 0} 4 + h = 4$$

(and say, "the limit as h approaches zero of $4 + h$ is 4")

base point and nearby points



Exact slope at $x = 2$ on
graph $y = x^2$ is 4

We can calculate the slope at any arbitrary point (x, x^2) on the graph $y = x^2$!

base point (x, x^2)

nearby point $(x+h, (x+h)^2)$

$$m_{\text{sec}} = \frac{\text{rise}}{\text{run}} = \frac{(x+h)^2 - x^2}{h}$$

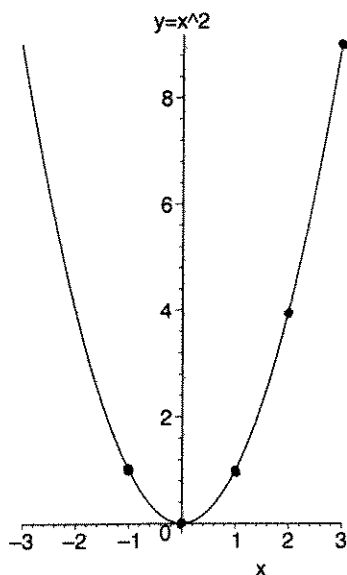
$$\lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \rightarrow 0} \frac{\cancel{x^2} + 2hx + h^2 - \cancel{x^2}}{h} = \lim_{h \rightarrow 0} 2x + h = 2x$$

Exercise 3 What is the slope of $y = x^2$ at $x = 1$?

at $x = 0$?

at $x = -1$?

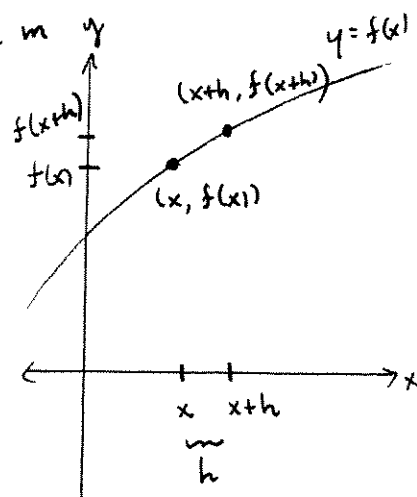
at $x = 3$?



Definition A: The graph of a function f is said to have slope m at a point $(x, f(x))$ on the graph provided

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = m$$

That is, provided the expression $\frac{f(x+h) - f(x)}{h}$ approaches m as h approaches zero. In this case we call the number m the derivative of f at x and denote it by $f'(x)$



Exercise 4 Find the derivative of the constant function $f(x) = c$. (a constant)
Explain why your answer makes sense

Exercise 5 Find the derivative of $f(x) = 5x - 7$.
Explain why your answer makes sense

Exercise 6 Find $f'(x)$ for $f(x) = 2x^2 + 5x - 7$