

P.1: Lines in the plane

slope of a line

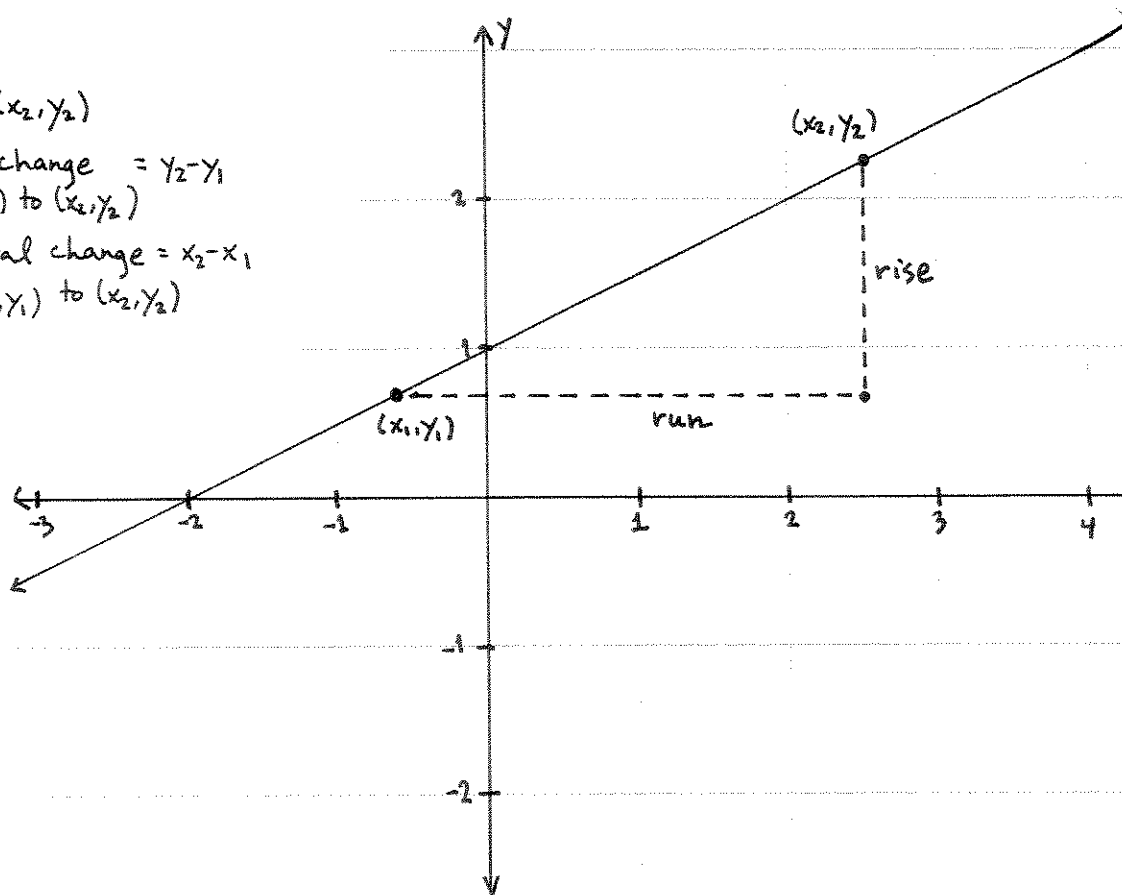
$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

between (x_1, y_1) and (x_2, y_2)

where $\text{rise} = \text{vertical change} = y_2 - y_1$
from (x_1, y_1) to (x_2, y_2)

$\text{run} = \text{horizontal change} = x_2 - x_1$
from (x_1, y_1) to (x_2, y_2)

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = m$$



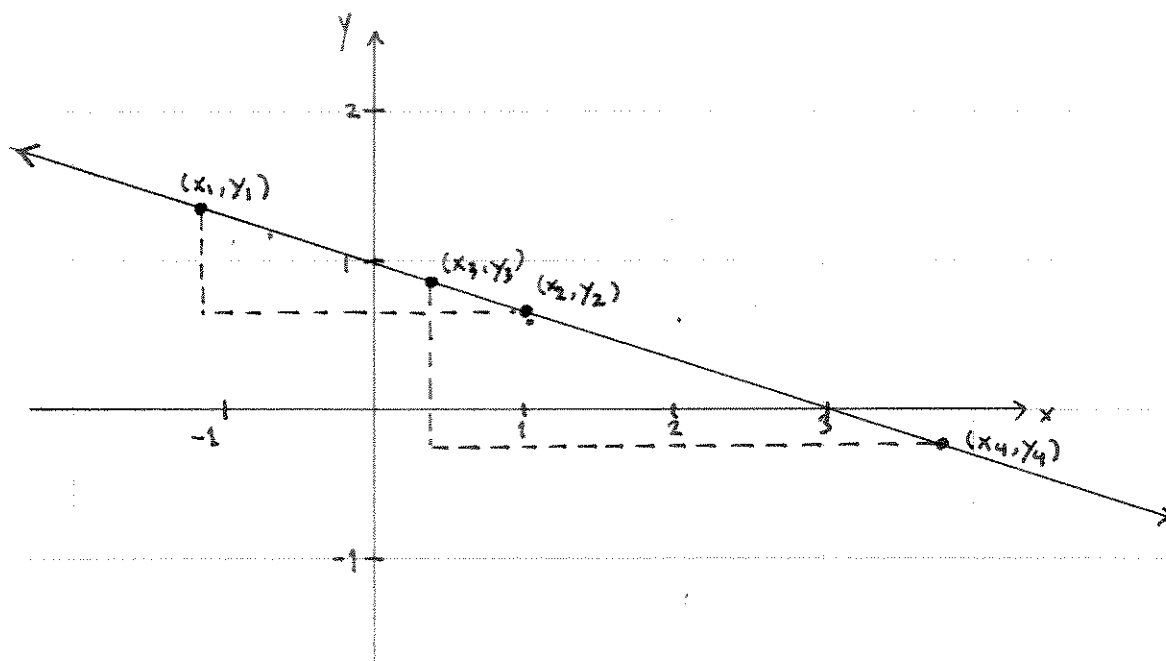
Exercise 1 Find m (slope) on the line above, from $(0, 1)$ to $(2, 2)$

Exercise 2 How about from $(2, 2)$ to $(0, 1)$

Exercise 3 Try other pairs of points on the same line. Is the slope always the same?

Exercise 4 What does $m > 0$ mean about the inclination of a line?
How about $m < 0$?

- Why value of slope is independent of choice of pair of points on a given line:
Use geometry! Hint: similar triangles



point-slope form of equation for a line through (x_0, y_0) with slope m
If (x, y) is a variable point on the line

$$m = \frac{y - y_0}{x - x_0}$$

yields

$$(y - y_0) = m(x - x_0)$$

Exercise 5: The point $(3, 0)$ is on the line above. What is the point slope equation of this line, using $(3, 0)$?

Exercise 6 Find an equation of the line through $(-1, -1)$ and $(1, 2)$, and sketch it on previous page.

slope-intercept form: If you simplify the point-slope form of the equation for a line (by solving for y), you get

$$y = mx + b$$

(where $b = y_0 - mx_0$)

this line intercepts the y -axis when $x=0$, and then $y = m \cdot 0 + b = b$
so b is the y -intercept

Exercise 7: Put the line equations from exercises 5, 6 into slope-intercept form, and compare to your sketches

General equation of a line Every equation of the form

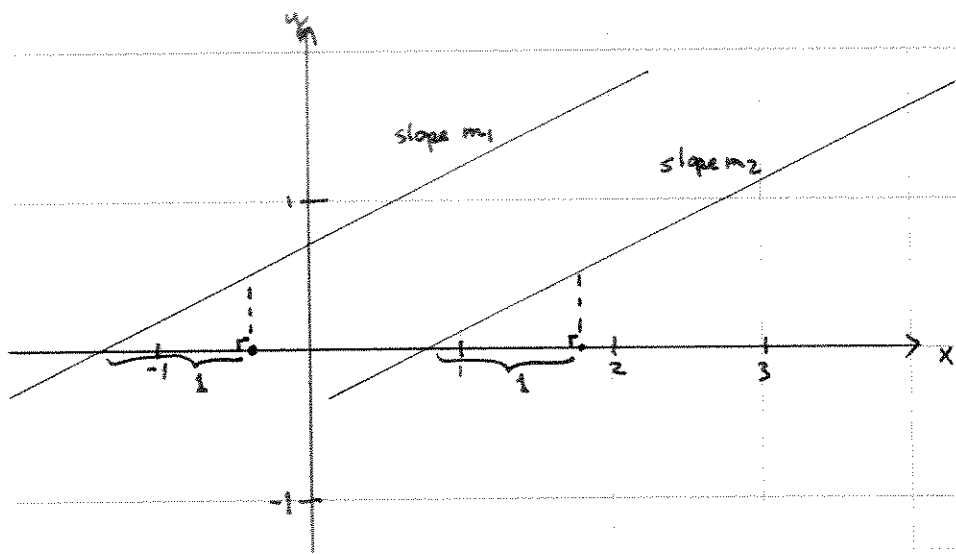
$$Ax + By + C = 0$$

is the equation of a line, and every line (even vertical ones) has an equation that can be put in this form

Exercise 8 Find the slope and y -intercept for the line with equation
 $2x + 3y - 6 = 0$

two lines are parallel if and only if they have the same slope

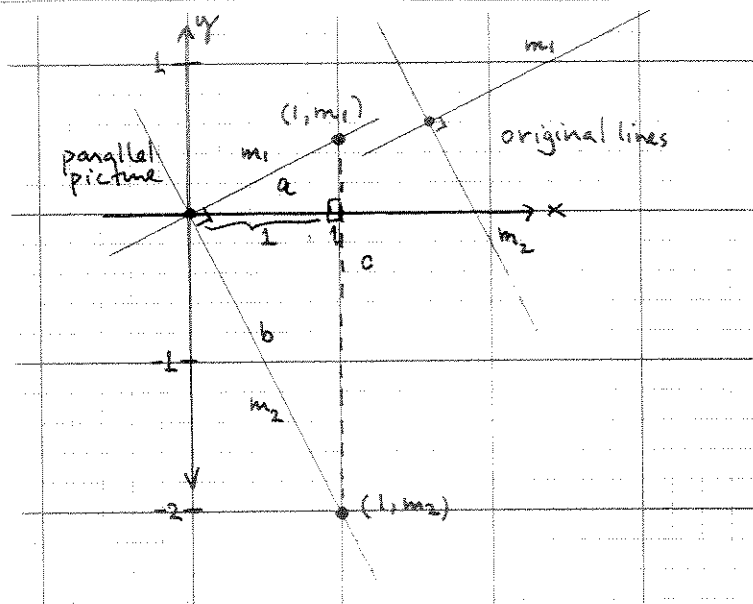
Details: (use congruent Δ 's)



Exercise 8: Find equation
of line $\parallel$ to page 2 line,
through point $(3, 1)$

two lines are perpendicular if and only if their slopes are "negative reciprocals",
i.e. $m_1 m_2 = -1$ ($m_2 = -1/m_1$)

Details: (Use Pythagorean Theorem)
 $a^2 + b^2 = c^2$



Exercise 9: Find equation of line $\perp$ to page 2 line, through $(3, 0)$.