Math 1210-3
Monday Jan 28

§1.1 Introduction to limits
(well, for us, it's a reintroduction, since we computed some limits in our polynomial calculus unit.)

$\lim_{x \to c} f(x) = L$ "the limit as x approaches c of f(x) equals L"

means, roughly speaking, that when x is "near" c (but $\neq c$), then f(x) is "near" L.

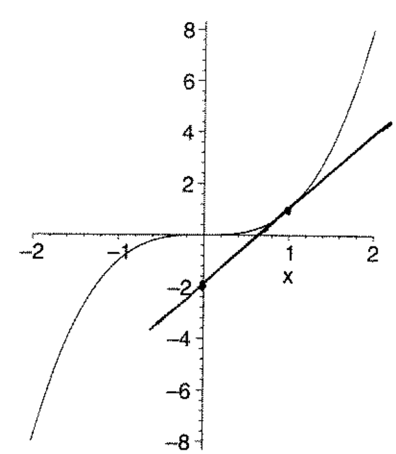
Exercise 1 : $\lim_{x \to 1} \frac{x^3 - 1}{x - 1} \overset{?}{=} \boxed{}$

1a) Use your calculator to fill in the table below, and guess the limit

x	f(x)
2	
0	
1.1	
.9	
1.01	
.99	

1b) Do long division on $\frac{x^3-1}{x-1}$, to "verify" your guess in (1a)

1c) This limit is actually the slope of the graph $y = x^3$, at $x = 1$. (Which you can find using polynomial calculus!) Why?



Exercise 2

Find $\lim_{t \to 2} \dfrac{t^3 - 2t^2 + 3t - 6}{t^2 + 3t - 10}$

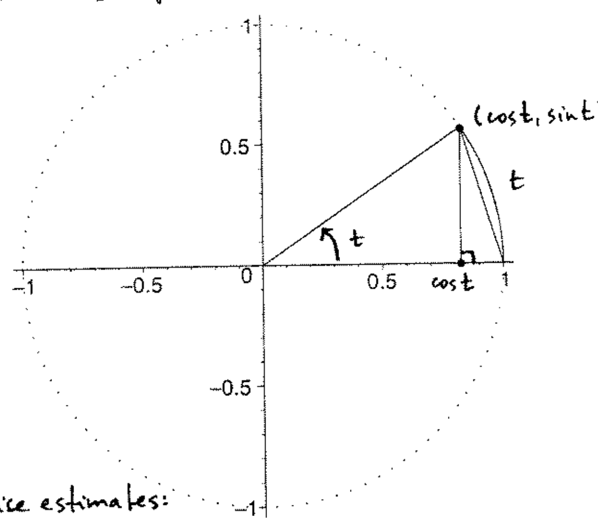
t	$f(t)$
2	$\frac{0}{0}$ X
2.01	

2a) guess:

2b) Use factoring and long division to deduce the exact value of this limit.

Exercise 3 (You need geometry, not algebra, to deduce these limits). (Unless specifically stated, the angle units for trig fns in calculus are always radians.)

3a) $\lim_{t \to 0} \cos t =$



3b) $\text{limit}_{t \to 0} \sin t =$

Precise estimates:

Using Pythagorean Thm and the fact that the shortest curve between two points is a straight line, deduce

$$0 \le (1-\cos t)^2 + \sin^2 t \le t^2$$

Now let $t \to 0$ and "squeeze"

Notice: You correctly answer these limit questions by just plugging in $t=0$, i.e. by computing $\cos(0)$, $\sin(0)$.

Functions for which $\lim_{t \to a} f(t) = f(a)$ are called continuous at $t=a$

Finish 0.6;

First complete page 3 Friday notes, Exercise 2.

Then,

<u>Exercise 4</u> (relates to identities on pages 47-48 Varberg)

4a) Why is $\cos^2 t + \sin^2 t = 1$ true?

4b) Use (4a) identity to show
$$1 + \tan^2 t = \sec^2 t$$
$$1 + \cot^2 t = \csc^2 t$$

4c) Derive the extremely important "addition angle" formulas
$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

How: Use the diagram below, to derive a formula which rotates points (a,b) an angle β about the origin. Then let $a = \cos\alpha$, $b = \sin\alpha$