

Math 1210-3

Wednesday Jan 23

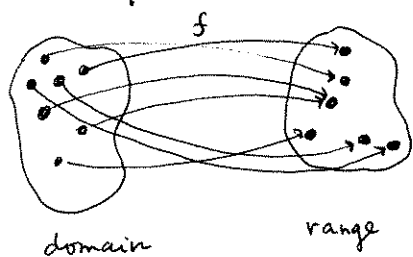
60.5-0.6

Functions, graphs of functions,
operations on functions

Definition A A function " f " is a rule of correspondence that associates with each object " x " in one set, called the domain, a single object $f(x)$ from the second set. The set of all the objects $f(x)$ so obtained is called the range of the function

Example : Let the domain be dairyman John's milk cows
Let $f(x)$ be the favorite pasture location of cow x .
Then the range is the collection of all favorite locations

schematic picture for a function:



Example Let the domain be the students in 1210-3 (YOU!)
Let $f(x)$ be student x 's final course grade.
Then the range is the actual grades that get assigned, from $A, A^+, B^+, \dots D^-, E, W$

But,
In Math 1210 our domain is (almost) always a subset of " x 's" in the real numbers
and the range is (almost) always a subset of " y 's" in the real numbers
(and in general, the domain elements are called independent variables,
and the range variables are called the dependent ones)
(since they are determined by the input x and the function f)

Exercise 1

Exercises 2,3 on page 3 Tuesday are about graphs of

functions, $y = f(x) = x^2 + 3x$

and $y = g(x) = 6 - 2x^2$

(which is a special case of
graphs of equations)

Do those two exercises, and figure out the domain and range of f and g .

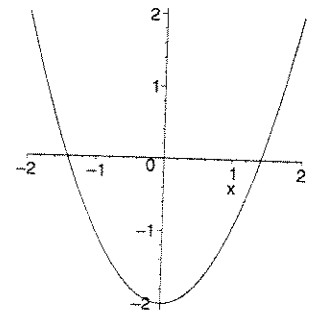
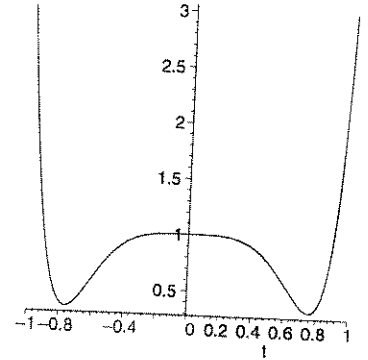
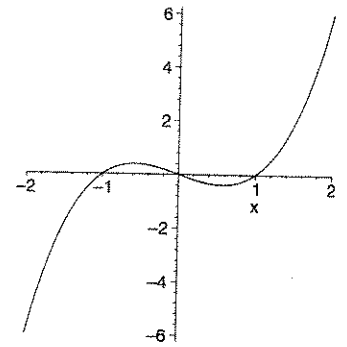
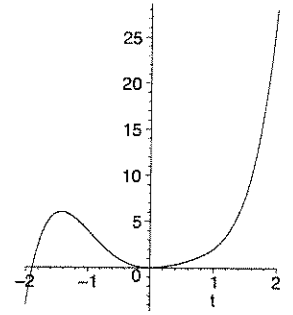
If a function formula is given without any domain specified, the domain is assumed to be the largest set possible, and called the natural domain

Exercise 2 Find the natural domains of

(a) $g(x) = x(50-x)$

(b) $f(t) = \sqrt{9-t^2}$

(c) $h(s) = \sqrt{-s} + \sqrt{s^2-16}$



A function is even if and only if $f(-x) = f(x)$

A function is odd iff $f(-x) = -f(x)$

Exercise 3 Are the following functions even or odd?

Why do you think the words even and odd were chosen for this concept? Match the functions to the 4 computer-generated graphs

(a) $f(x) = x^2 - 2$

(b) $g(x) = x^3 - x$

(c) $h(t) = 6t^8 - 4t^4 + 1$

(d) $k(t) = t^5 - 2t^3 + 3t^2$

Exercise 4 An even function $f(x)$ has graph $y = f(x)$ which is symmetric with respect to _____

An odd function $g(x)$ has graph $y = g(x)$ which is symmetric with respect to _____

Let $f(x)$, $g(x)$ be functions.

Then the "new" functions

$$(f+g), fg, f/g, fog$$

$\uparrow$ $\uparrow$ $\uparrow$ $\nwarrow$
 sum product quotient composition

are defined by

$$(f+g)(x) = f(x) + g(x)$$

$$(fg)(x) = f(x)g(x)$$

$$(f/g)(x) = f(x)/g(x)$$

$$(fog)(x) = f(g(x))$$

Exercise 5 Let $f(x) = x^3 - x$ (picture of graph $y = x^3 - x$ shown below.)

$$g(x) = x - 4$$

$$h(x) = 2x$$

$$k(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Compute, and graph these functions:

a) $(f \circ g)(x)$

b) $k(x)$

c) $(h+k)(x)$

d) $(hk)(x)$

e) $\frac{h}{k}(x)$

f) $(h \circ f)(x)$

g) $(f \circ h)(x)$

h) $(k \circ g)(x)$

