Math 1210-3
Wednesday Jan 16

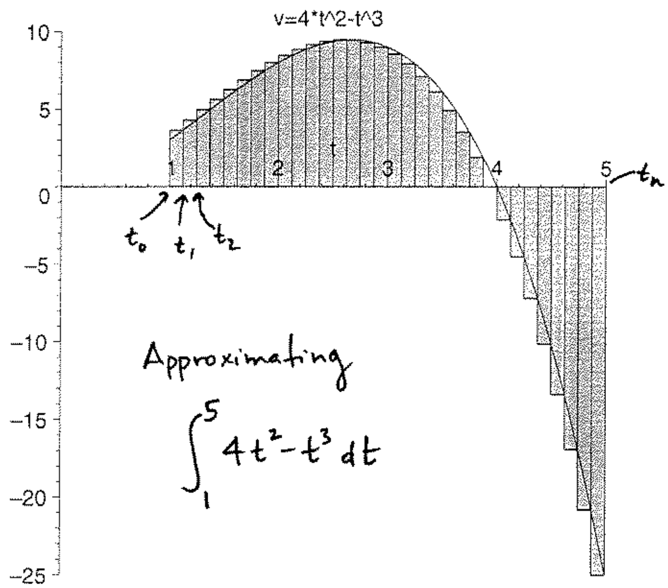
- Friday quiz is sections P.3 – P.5
  solutions to V.I.P. will be posted

We begin today with the discussion of F.T.C. on pages 4-5 Tuesday.
After that, we'll discuss one way to intuitively understand this amazing theorem. My guess is that the explanation I'll give is how Newton thought about it. (This is not the poly calc notes explanation.) We will cover these concepts in much greater detail later in our course, in §4.3-4.4 Varberg, so if this particular concept seems a bit confusing now, it should get cleaned up later

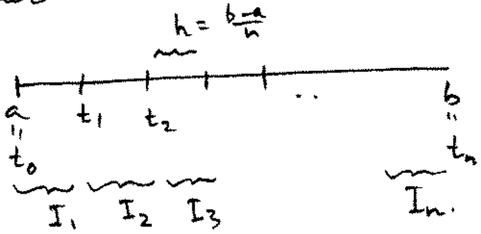
Why FTC is true (?!)

Compare the picture on the right to the one on p. 4 Tuesday – they're the same, except we've labeled the horizontal axis t (e.g. seconds) & vert axis "v" for velocity, e.g. ft/s.

So although we may have started with a graph $y=f(x)$, we are now considering an object with velocity $f(t)$, and its graph $v=f(t)$ ("same" function). This object has a corresponding position function $s(t)$.



- Consider the subdivision of the interval $a \le t \le b$



- Consider a competing car that travels in fits and starts, traveling at constant velocity $f(t_i)$ for all of the time interval I_i. For example, its velocity graph could correspond to the "step" function graph on the right. This car has position function too, say $\tilde{s}(t)$.

$$\begin{aligned}
&\underbrace{f(t_1)}_{\text{const vel}}\underbrace{h}_{\text{time}} = \underbrace{\tilde{s}(t_1)-\tilde{s}(t_0)}_{\text{net displacement}}\\
+\;& f(t_2)h = \tilde{s}(t_2)-\tilde{s}(t_1)\\
+\;& \qquad\vdots \qquad = \tilde{s}(t_3)-\tilde{s}(t_2)\\
+\;& f(t_n)h = \tilde{s}(t_n)-\tilde{s}(t_{n-1})\\
\hline
&\sum_{i=1}^{n} f(t_i)h = \tilde{s}(t_n)-\tilde{s}(t_0)\\
&\qquad\qquad = \tilde{s}(b)-\tilde{s}(a).
\end{aligned}$$

the Riemann sum for f → $\sum_{i=1}^{n} f(t_i)h$

(over).

- Thus the Riemann sum for the "fits and starts" car yields the net displacement in its location for $a \le t \le b$, i.e.

$$\tilde{s}(b) - \tilde{s}(a).$$

As $n \to \infty$ $(h \to 0)$, the limiting net displacements of the "fits & starts" car

$$\tilde{s}(b) - \tilde{s}(a)$$

will approach

$$s(b) - s(a)$$

as long as the velocity $f(t)$ is continuous, because when h is small each car is traveling at nearly the same velocity.

i.e. $$\int_a^b f(t)\,dt = \lim_{h \to 0} \sum_{i=1}^{n} f(t_i) h = s(b) - s(a).$$

But $\underset{\substack{\uparrow \\ \text{position}}}{s(t)}$ is an antiderivative of $\underset{\substack{\uparrow \\ \text{velocity.}}}{f(t)}$

So if $F(t)$ is any other antideriv, then

$$F(t) = s(t) + C$$

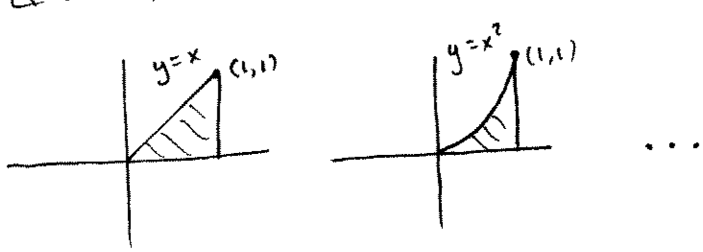
so $$F(b) - F(a) = s(b) + C - (s(a) + C) = s(b) - s(a).$$

i.e. $$\boxed{\int_a^b f(t)\,dt = F(b) - F(a)}$$

Hopefully this makes sense! It <u>is</u> magic that you can also apply it to compute areas. If, instead, it just seems very confusing, worry mostly about being able to do the P.S problems, for now.

<u>Exercise 1</u> For $n = 0, 1, 2, 3, \ldots$

find the area between $y = x^n$, & the x-axis, for $0 \le x \le 1$



<u>Exercise 2</u> If the velocity of a car is $v(t) = -30t^2 + 15t + 45$ miles/hour

for $0 \le t \le 2$ hours,

and the car is driving along a marked hiway, then

2a) What is the <u>net</u> distance traveled between $t=0$ and $t=2$ hours?

2b) What is the <u>total</u> distance traveled?

$v = -30t^2 + 15t + 45$