Math 1210-3
Monday Jan 14
P.3 - P.4

Physics interpretation of derivative:

We defined

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

and interpreted it as the slope of a graph.

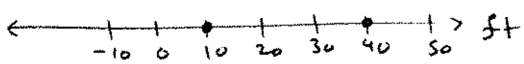
There are completely different interpretations of the same definition.

Suppose $s(t)$ is the location of an object a long a number line at time t. (The number line might be marked off in feet, and time might be measured in seconds.... or you might use miles and hours.)

Then $\dfrac{s(t+h)-s(t)}{h} = \dfrac{\text{displacement}}{\text{time}}$ $\quad \frac{ft}{sec}$ (or $\frac{miles}{hour}$)

is called <u>average velocity</u>

<u>Exercise 1</u>: If $s(0)=10$, $s(2)=40$ $\quad$ ($s(t)$ feet at time t sec)



What is the average velocity between $t=0$ and $t=2$ sec.?

So, it makes sense to define $\boxed{s'(t) = v(t) \text{ as the } \underline{\text{velocity}}}$ at time t, since

$$\lim_{h\to 0} \frac{s(t+h)-s(t)}{h} = s'(t)$$

is the limit of average velocities over shorter & shorter time intervals.

<u>Exercise 2</u>: Most of you know of a electro-mechanical device which computes derivatives very frequently. What is it?

Similarly $v'(t) = \lim_{h\to 0} \dfrac{v(t+h)-v(t)}{h}$ $\quad$ ($\frac{ft/sec}{sec}$ or ft/sec^2)

measures how fast velocity is changing and is called acceleration $a(t)$.

$$\boxed{v'(t) = a(t).}$$

Exercise 3: If a ball is dropped off the top of a 64 ft. high building, then its height $s(t)$ seconds later is described by the formula

$$s(t) = -16t^2 + 64 \qquad \text{ft.}$$

3a) When does it hit the ground?

3b) What is its velocity when it hits?

3c) What is the acceleration function?

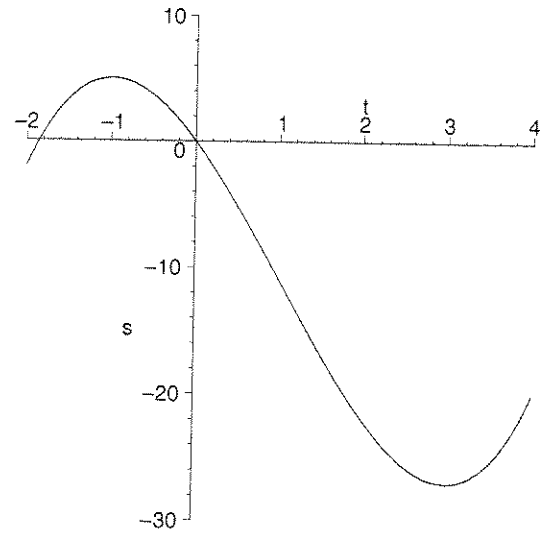
Putting the two interpretations of derivative together:

Suppose an object is moved along a number line so that its position is

$$s = t^3 - 3t^2 - 9t \quad \text{meters}$$

at time t seconds.

plot of $s = t^3 - 3t^2 - 9t$



Exercise 4a) What is the object's velocity at $t = 2$ sec.?

Exercise 4b) What is the slope of the graph $s = t^3 - 3t^2 - 9t$ ft, at $t = 2$ sec.?

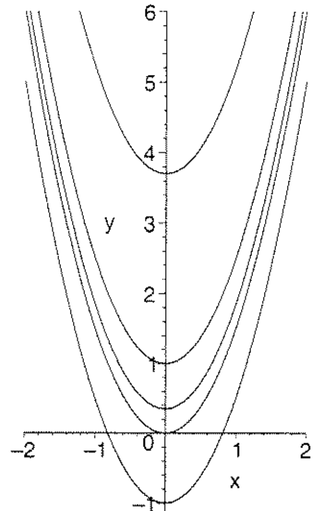
Exercise 4c) Sketch the tangent line to the graph $s = t^3 - 3t^2 - 9t$, and see if your sketch is consistent with 4ab

Exercise 4d) At what t values does the object turn around? Explain. Relate this discussion to the velocity function.

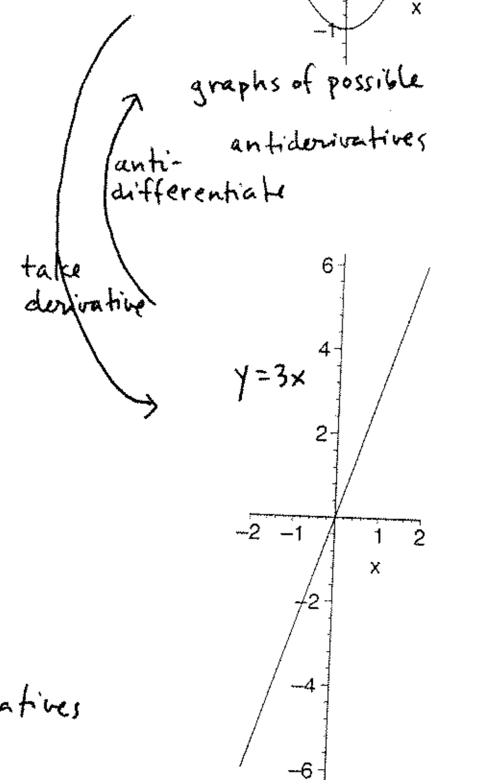
§P.4 Antiderivatives of functions
(also called antidifferentiation)

<u>Definition A</u> If $f(x)$ is a function, then an <u>antiderivative</u> for f is a function having $f(x)$ as its derivative

(This is the reverse procedure to taking derivatives.)



<u>Exercise **5**</u>: Using your knowledge of derivatives and working backwards, find all the antiderivatives of $f(x) = 3x$ that you can find.
The pictures to the right are a hint.



<u>Definition B</u> If $f(x)$ is a function, the set of antiderivatives for $f(x)$ is denoted

$$\int f(x)\,dx$$

and is called the <u>indefinite integral of $f(x)$</u>. (<u>with respect to x</u>)

Note, the "x" in "dx" says that x is your variable.

<u>Exercise **6**</u>

$$\int 3x\,dx =$$

(You will rely on the theorem at the top of the next page to answer this exercise completely)