

Name _____

Student I.D. _____

Math 1210-3
Final Exam
April 30, 2008

Please show all work for full credit. This exam is closed book and closed note, except for the single customized 4 by 6 inch index card you have been allowed to bring. (Two Varberg formula pages are at the end of the exam.) You may also use a scientific calculator, but not a graphing calculator or one which can compute derivatives and integrals. There are 150 points possible, as indicated below and in the exam. You have two hours to complete the exam, so apportion your time accordingly. Good Luck!!

Score	POSSIBLE
1_____	20
2_____	15
3_____	20
4_____	15
5_____	15
6_____	10
7_____	15
8_____	30
9_____	10
TOTAL_____	150

1) Compute the following limits. Limits which equal infinity (or -infinity) are possible. It is also possible that the limit does not exist, in which case you should answer DNE, and indicate your reasoning.

$$1a) \lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x^3 - 27}$$

(5 points)

$$1b) \lim_{x \rightarrow 2^+} \frac{x + 8}{x^2 - 4}$$

(5 points)

$$1c) \lim_{x \rightarrow 0} \frac{3x}{|x|}$$

(5 points)

$$1d) \lim_{h \rightarrow 0} \frac{(5+h)^3 - 125}{h}$$

(This is a derivative in disguise, so a short cut is available!)

(5 points)

2) Compute the following derivatives:

2a)

$$D_t \left(\pi^2 + 3t^2 - \frac{6}{t} \right)$$

(5 points)

2b)

$$D_x [\sec(x)^2 - \tan(x)^2]$$

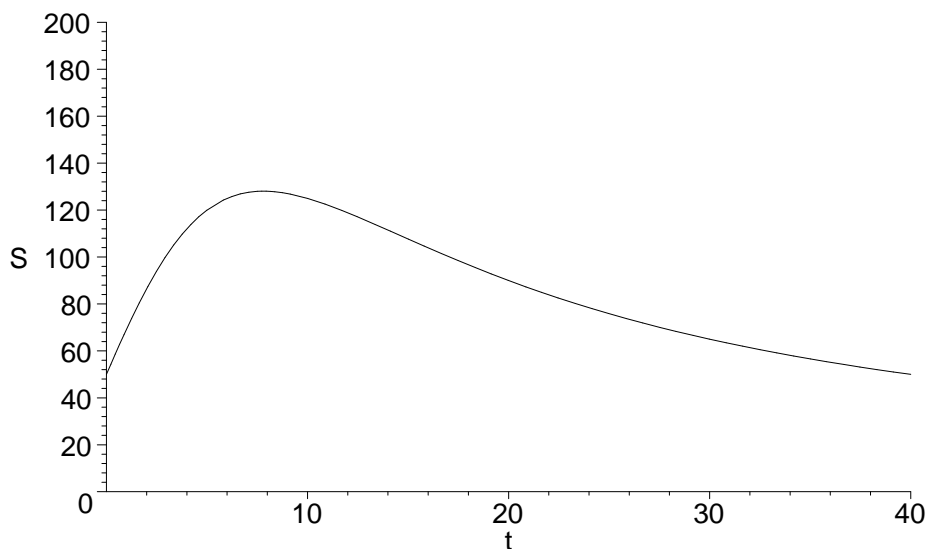
(5 points)

2c) $f'(x)$ for

$$f(x) = \frac{1}{\sqrt{x^3 + 4}}.$$

(5 points)

3) Mathville is hit with a spring flu epidemic! The number $S(t)$ of sick people at time t days after the epidemic begins is plotted below:



3a) Using the graph above, estimate how many people are sick on day $t=20$. (It might be helpful to fold your paper and use the edges to draw any lines you need.) Show work!
(5 points)

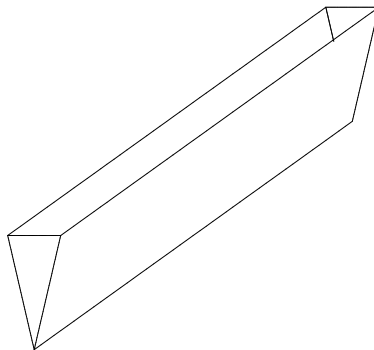
3b) Using the graph above, estimate how fast the number of sick people is decreasing on day $t=20$. Show work!
(5 points)

3c) It turns out the sick people graph shown above is very close to the graph of the function

$$S(t) = \frac{20t + 50}{1 + 0.01t^2}.$$

Use this formula for $S(t)$ to calculate values for how many people are sick on day $t=20$, and also to calculate how fast the number of sick people is decreasing on day 20. (You should get numbers close to the ones you found geometrically in parts (3a) and (3b) !)
(10 points)

4) A drinking trough for cattle has a cross section in the form of an inverted isosceles triangle. The base of the triangle is 2 feet long and its height is 3 feet, as shown below. The trough is 20 feet long.



If water is filling the trough at a constant rate of 4 cubic feet per minute, then how fast is the water depth increasing when the depth is 2 feet?

(15 points)

5) Use Calculus to find the point on the line $2x + y = 6$ which is closest to the origin. (Hint: minimize the square of the distance from (x,y) to the origin.)

(15 points)

6) The following table shows the depth of a river at a certain cross-section perpendicular to the water flow. The river is 20 feet wide at this point. The depths y in feet, at position x feet from one shoreline are given by

x (location)	0	5	10	15	20
y (depth)	0	3	6	4	2

6a) What is the Simpson's (parabolic) rule approximation to the cross-sectional area of the river at the location above, with $n=4$ subdivisions of x -interval $[0,20]$?

(5 points)

6b) Use your answer to part (6a) to approximate the rate at which water is flowing down the river, assuming that the water velocity at this cross-section is approximately two feet per second. Make sure to include your units as part of your answer.

(5 points)

7) Compute the following integrals:

$$7a) \int_{-4}^4 4x - 12x^2 + 5 \sin(3x) dx$$

(5 points)

$$7b) \int_0^1 x(3x^2 - 1)^2 dx$$

(5 points)

$$7c) \int_0^{\frac{\pi}{8}} 10 \cos(4t) dt$$

(5 points)

8a) Sketch the region between the graphs of $y = x^2$ and $y = 9$. Make sure to label the curves and the points where they intersect.

(5 points)

8b) Find the area of the region you sketched in 8a).

(5 points)

8c) Use planar slabs or cylindrical shells (your choice!) to find the volume of revolution obtained by rotating the region in (7a) about the vertical line $x = -3$.

(15 points)

8d) Check your answer in 8c), by using Pappus' Theorem to recompute the volume of revolution.

(5 points)

9) The parametric curve

$$x = \cos(t)$$

$$y = \sin(t)$$

describes the upper half of the unit circle, for $0 \leq t \leq \pi$. When this curve is rotated about the x-axis it generates a sphere of radius 1. Use a definite integral for surfaces areas of revolution, to verify that this sphere has area 4π .

(10 points)