Math 1210-3
Friday Feb. 8

(Finish p.2 Wed.)
Now that you've been convinced that "rates of change" are everywhere...

§2.2-2.3 : Differentiability; sum, product, quotient rules.

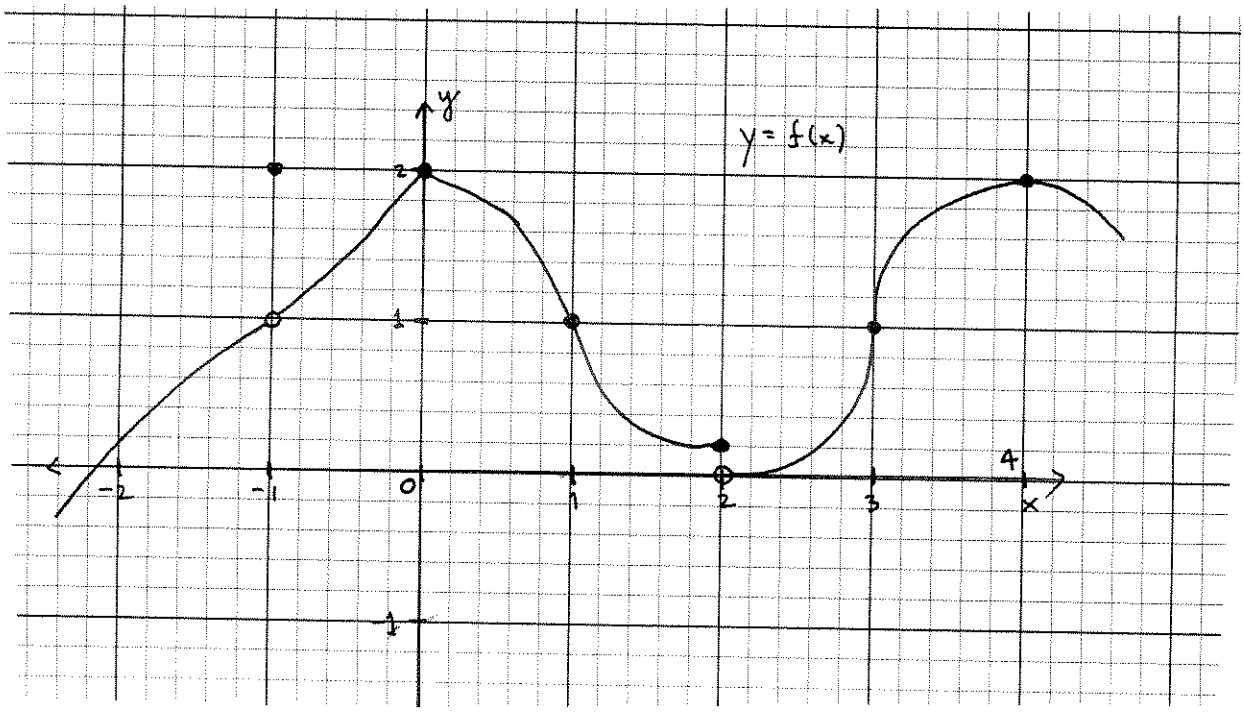
f is differentiable at $x=c$ means

$$f'(c) = \lim_{h\to 0} \frac{f(c+h)-f(c)}{h} \text{ exists.}$$

Sometimes we substitute $x = c+h$, $h = x-c \to 0$, i.e. $x \to c$:

$$f'(c) = \lim_{x\to c} \frac{f(x)-f(c)}{x-c}$$

Exercise 1 Decide from the graph below whether $f'(c)$ exists at the indicated x-values – use the geometric meaning of the limit definition of derivative. If the derivative exists, estimate its value.



$f'(-1)$

$f'(0)$

$f'(1)$

$f'(2)$

$f'(3)$

$f'(4)$

Relationship between continuity and differentiability (use Exercise 1 to visualize this)

(a) If $f(x)$ is differentiable at $x=c$, then f is continuous at $x=c$.

(b) If $f(x)$ is continuous at $x=c$, it may or may not be differentiable there.

proof of (a): Let $f(x)$ be differentiable at $x=c$.

Write

$$f(x) = f(c) + (f(x) - f(c))$$

$$f(x) = f(c) + \left(\frac{f(x)-f(c)}{x-c}\right)(x-c)$$

so $\lim_{x \to c} f(x) =$

Exercise 2 Find a function in our "zoo" which shows (b), i.e. continuity does not imply differentiability.

Exercise 3 Use the limit definition of derivative to find $f'(x)$, for for $f(x) = \frac{2}{x+7}$

Life is easier if you know...

Differentiation rules theorem

(1) $(f+g)' = f' + g'$ sum rule

(2) $(cf)' = cf'$ (c constant) constant multiple rule.

} we showed these in polynomial calc. – do you remember how?

(3) $(fg)' = f'g + fg'$ product rule

(4) $(f/g)' = \frac{f'g - fg'}{g^2}$ quotient rule

(there's a rule for differentiating compositions too, it's called the chain rule, §2.5)

Exercise 4 redo exercise 3 with the quotient rule

Exercise 5 if n is a positive integer, use the quotient rule to find $f'(x)$ for $f(x) = x^{-n} = \frac{1}{x^n}$

Exercise 6 What is $[(x^3+6x)(2x^2+5)]'$? Check ans. by multiplying out first.

Exercise 7 Find the derivative of $\tan x = \frac{\sin x}{\cos x}$.

Exercise 8 The product rule and quotient rule seem mysterious, but we can prove them!

(a) Show, using the limit definition, that

$$(fg)' = f'g + fg'$$

(b) If we have time, try $\left(\frac{f}{g}\right)'$! (If not, see page 112 of text).