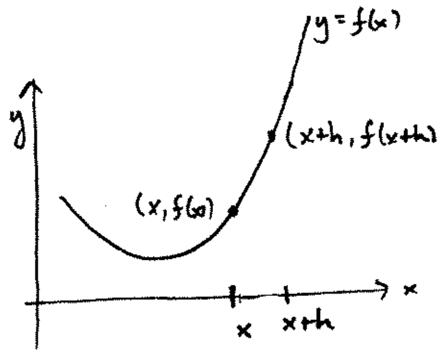
Math 1210-3
Wednesday Feb 6

§2.1 Derivatives and rates of change.



We know the definition for derivative!

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

We call $\frac{f(x+h)-f(x)}{h}$ the average slope between x and x+h
or
the average rate of change of f(x), between x and x+h

the derivative $f'(x)$ is the limit of these average slopes (or rates of change), and is sometimes called the instantaneous rate of change of f with respect to x

We know this sort of specific example!

The acceleration of gravity on the moon is about 1.6 meters per second per second, so if a rock is thrown vertically (are you tired of only studying dropped objects, or vertically thrown ones?), with initial velocity of 16 meters per second, and initial height, the height function at time t seconds later will be given by

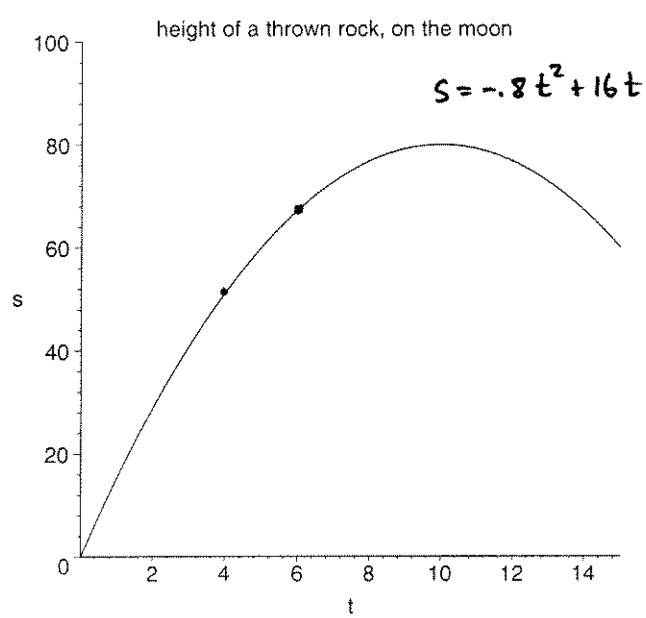
$$s(t) = -0.8\, t^2 + 16\, t.$$

Here is a graph of the height function, for the first 15 seconds:

Exercise 1 (a) Find the average rate of change of height ("average velocity") between t=4 and t=6 seconds. Include units!

b) Calculator! average velocity between t=4 & t=4.1 sec.?

1c) What is the velocity at t=4 sec,? i.e. the instantaneous rate of change of position?



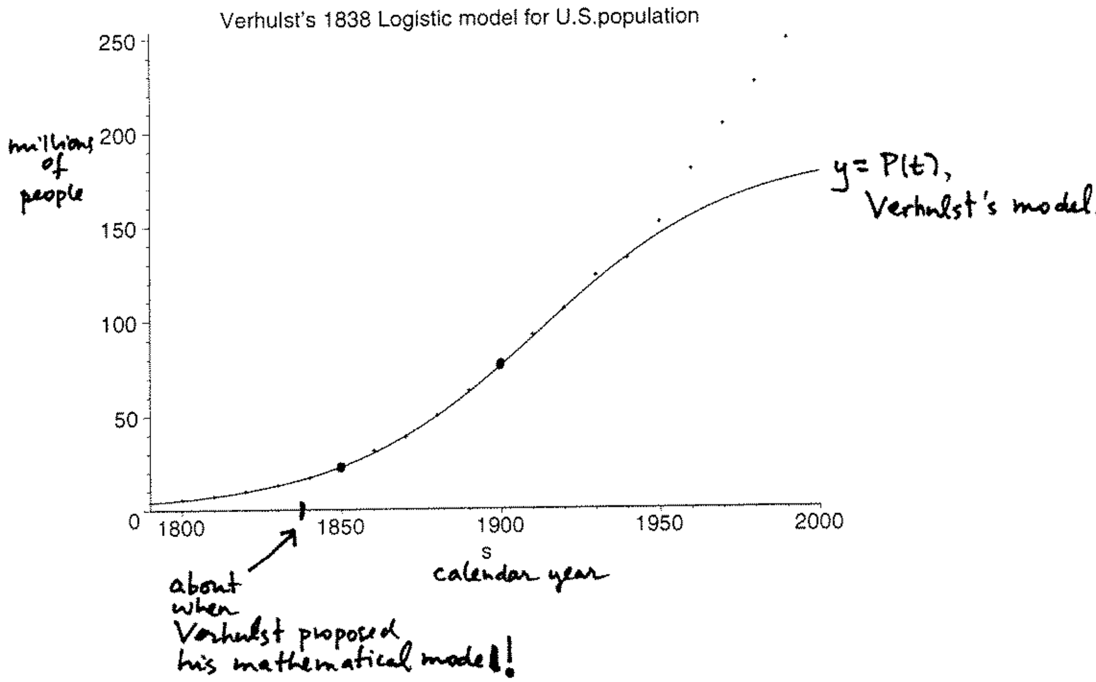
Derivatives = rates of change, cont'd:

In the 1830's a Belgian Demographer Pierre Verhulst proposed that many populations follow what is now called the logistic model of population growth:

$$P'(t) = r\,P(t)\left(1 - \frac{P(t)}{k}\right).$$

He found good fits for the rate parameter r, and the "carrying capacity" k, based on ~~population~~ U.S. population data from several preceeding decades, and then found a solution function to solve this equation. That led to predictions for future U.S. population numbers. This example illustrates the effectiveness and limitations of mathematical modeling, especially for population problems!

The vertical axis units are millions of people. The horizontal axis units are calendar years.
points represent actual data. Smooth graph is Verhulst's prediction.



Exercise 2a) Use the graph and data above to estimate the average rate of change of population ("population growth rate") between 1850 and 1900. Include units! (It may help you to fold your paper.)

2b) Estimate the exact population growth rate in 1900. (Use tangent line!)

2c) What happened around 1950 (1945-) that messed up Verhulst's model, which had been amazingly accurate until then?

Global warming/cooling. (A topical question involving rates of change)

As a scientist, or scientifically trained person, you can chase down the best current science – which can change!

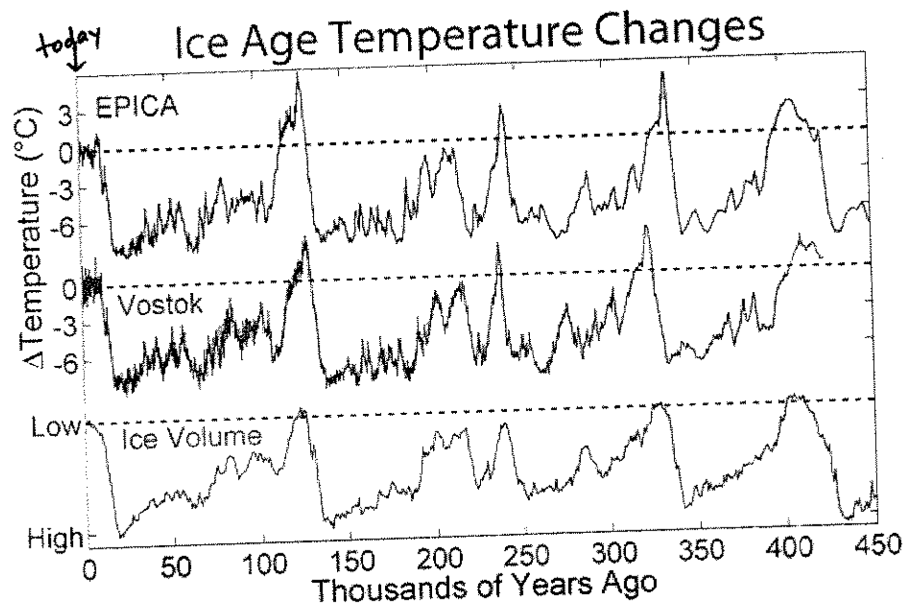
Beware – no matter what you "believe" – the "common wisdom" can sometimes be wrong.

All plots on this page are from http://en.wikipedia.org/wiki/Global_warming
The international science consensus on climate change can be found at http://www.ipcc.ch
(Intergovernmental panel on climate change – they won the Nobel Peace Prize in December

In the long-term sense, (tens of thousands of years), global cooling is probably a greater threat than global warming, e.g. large portions of currently habitable land are covered with glaciers during ice ages:

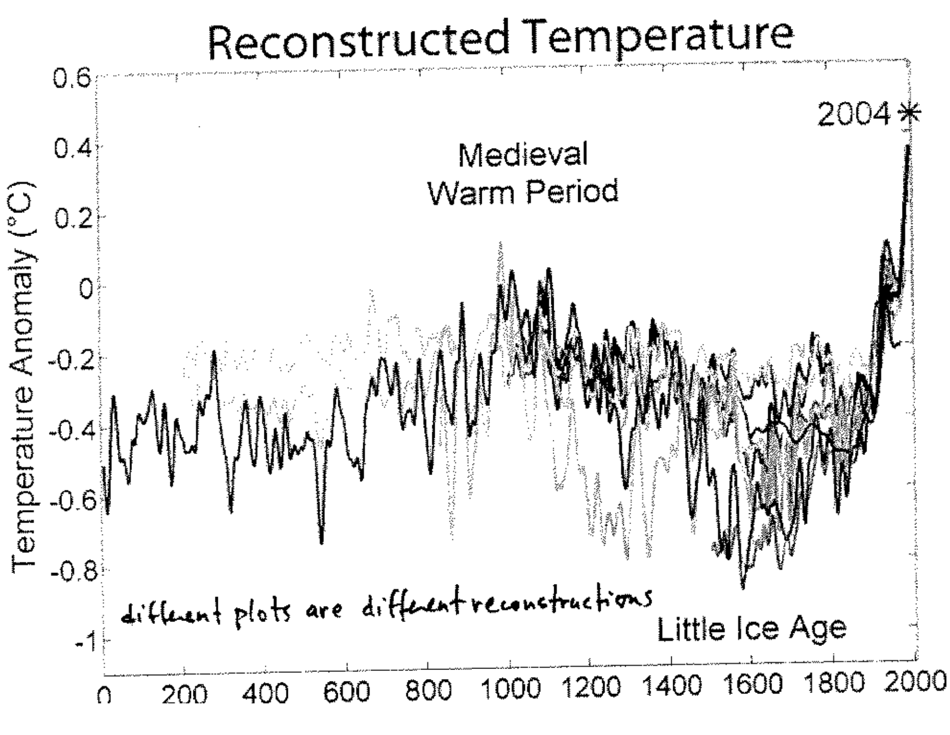
the last 10,000 years – which is the time period in which humans took over the earth – has been a warm period!

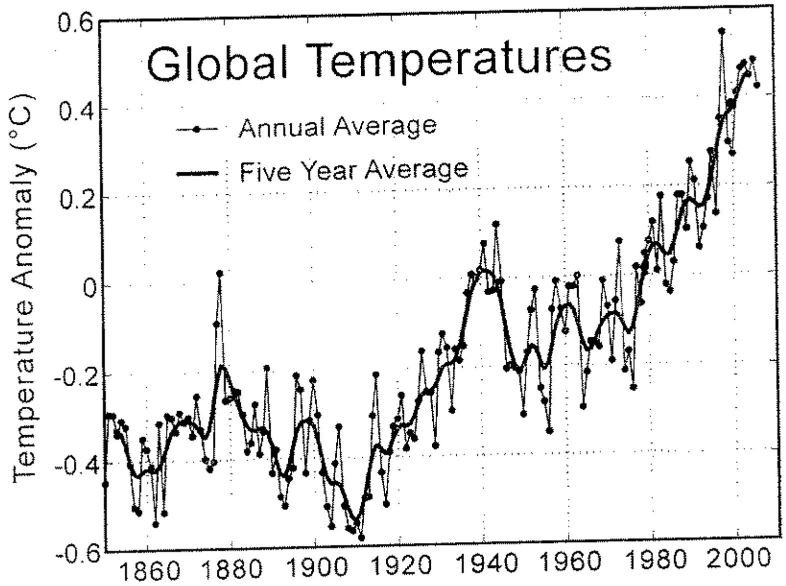
$\boxed{\text{what could be causing changes on these time scales?}}$



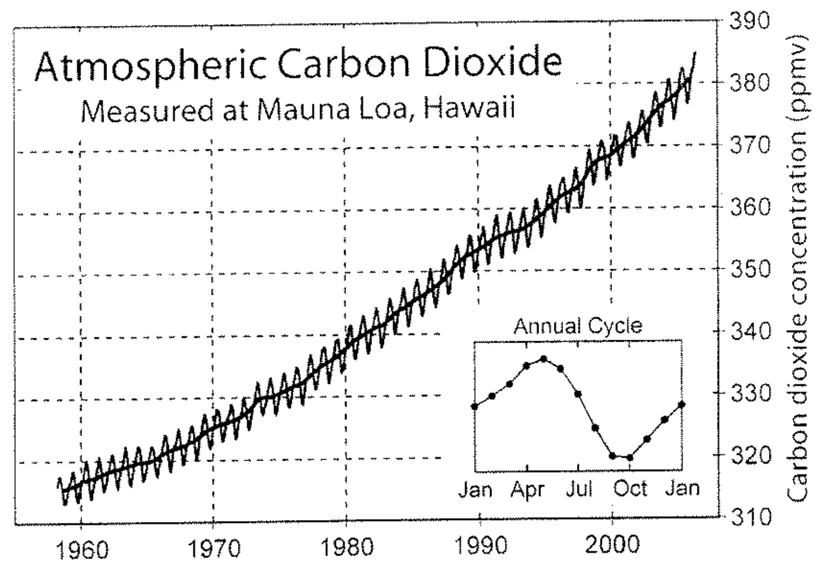
the last 2000 years...

$\boxed{\text{what could be causing changes on these time scales?}}$



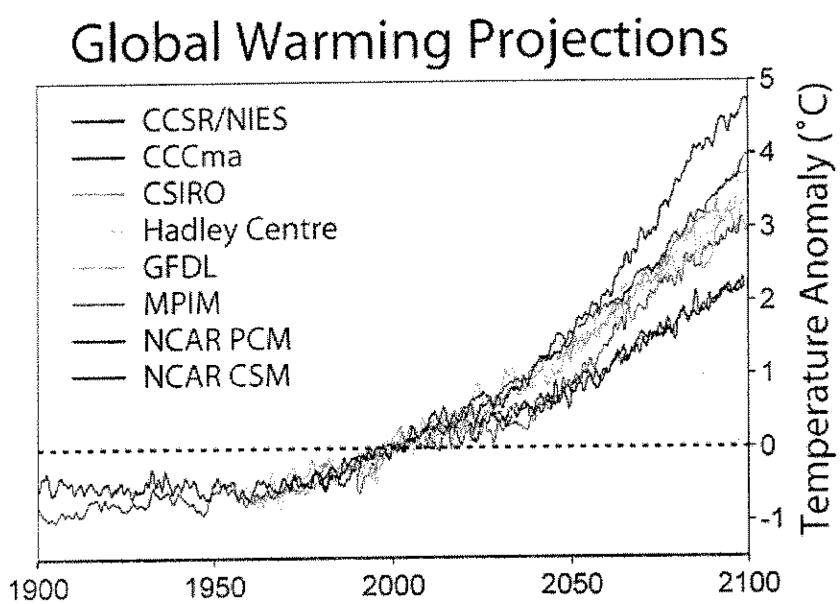


Exercise 3 Using the "5-year average" curve, estimate how fast the average temperature (globally) was rising between 1980 and 2000. Include units!!



Exercise 4 About how fast was the amount of atmospheric carbon dioxide increasing between 1990 and 2000? Include units!!

oncentration estimates for
.100 are 541-970 ppm.)



The dynamical system of earth's climate involves many variables, and is extremely difficult to model. As scientists collect more data, the models should improve. (Also, human impact may vary.)

If you like math and are interested in global warming, one of our Professors, Ken Golden, is giving the Math Dept. Colloquium this Thursday, Feb 7, here (JWB 335), @ 4:15 p.m.
www.geotimes.org/nov07/article.html?id=nn_seaice.html