Math 1210-3
Tuesday Feb 5

Finish last page of Monday's notes.
(Maybe you want to talk about some webworks problems first?)
§1.6 continuity.

One-sided limits

$\lim_{x \to c^-} f(x) = L$ means as x approaches c, but $x < c$, i.e. x approaches c from the negative, or "left" side, then, $f(x)$ approaches L.
Could you say this in the ε-δ language?

$\lim_{x \to c^+} f(x) = L$ means $f(x)$ approaches L, as x approaches c from the positive (right) side.

Exercise 1: Let the graph of $y = f(x)$ be as indicated.

Then answer:

1a) $\lim_{x \to -1^-} f(x) =$

b) $\lim_{x \to -1^+} f(x) =$

c) $\lim_{x \to -1} f(x) =$

d) $\lim_{x \to 2^+} f(x) =$

e) $\lim_{x \to 2^-} f(x) =$

$\lim_{x \to c} f(x) = f(c)$

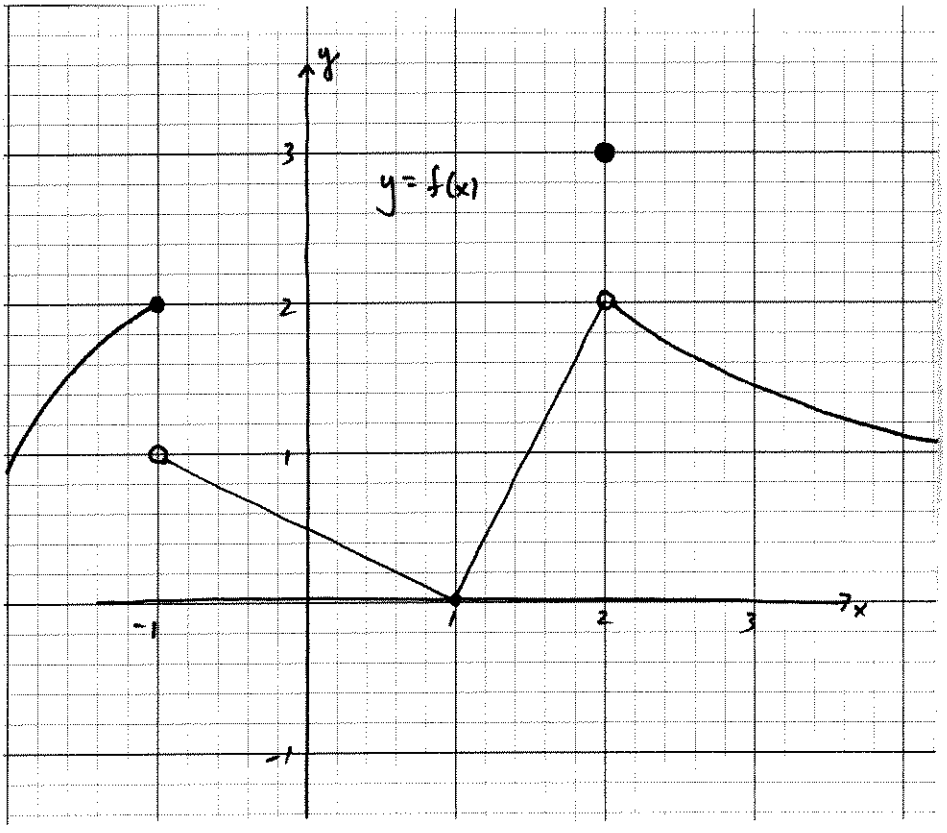
f) Discuss the continuity of f at

$x = -1$

$x = 1$

$x = 2$

$x = 1/2$



Theorem $\lim_{x \to c} f(x) = L$ if and only if both $\lim_{x \to c^-} f(x) = L$ and $\lim_{x \to c^+} f(x) = L$

(So f is continuous at $x = c$ iff $\lim_{x \to c^-} f(x) = f(c)$ and $\lim_{x \to c^+} f(x) = f(c)$.

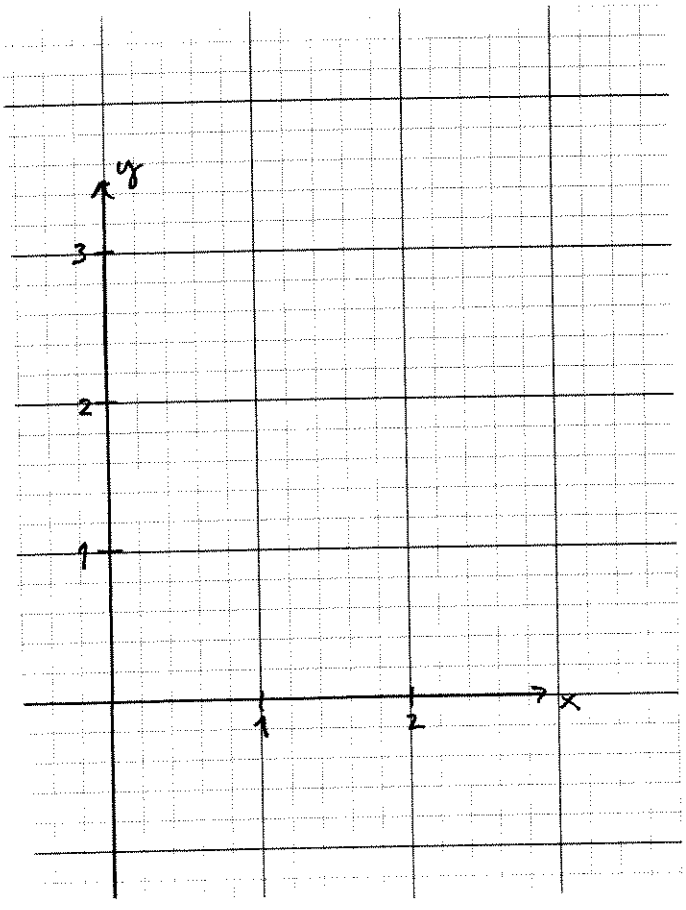
If only one of these limit statements is true, f is called continuous from the left, or continuous from the right.

Exercise 1g). Re-discuss the continuity of f at $x = -1$, on page 1

Exercise 2 Let $f(x) = \begin{cases} x^2 + 1 & x \leq 1 \\ -x + 3 & x > 1 \end{cases}$

(a) Sketch the graph of f

(b) Is $f(x)$ continuous at $x = 1$?



Exercise 3 Let $g(x) = \begin{cases} (x-1)^2 + a, & x < 2 \\ x - 2 & x \geq 2 \end{cases}$

For what value of a is $g(x)$ continuous at $x = 2$?

Exercise 4 : Let $f(x) = \dfrac{x^3-3x^2+2}{x^2-1}$

(a) What is the natural domain of f?

(b) can you define $f(1)$ so that $f(x)$ is continuous at $x=1$?

(c) Can you define $f(-1)$ to make $f(x)$ continuous at $x=-1$?

Exercise 5. Let $f(x) = \begin{cases} \dfrac{|x|}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$

(a) Is $f(x)$ continuous at $x=0$?

(b) Sketch $y = f(x)$