Math 1210-3
Monday Feb. 4

- I'll try to get to JWB 335 early, so you can pick up your graded exams before class. The solutions are posted on our website, along with the score distribution (which is also at the end of today's notes).
- You can use your exam score as an (imperfect) diagnosis of your current understanding of the course material. If you scored near or below 50% it is quite likely you are not understanding large portions of our material. If this is the case you need to take assertive action if you wish to learn the material and pass the class.... study groups, tutors, effective studying techniques, can all help. Other (links to) suggestions are posted on our web page. Don't avoid or delay taking action!!

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§1.4: trig limits

<u>Exercise 1</u>: Use

$$\left.\begin{aligned}\lim_{t\to 0} \sin t &= 0 \qquad (=\sin 0)\\ \lim_{t\to 0} \cos t &= 1 \qquad (=\cos 0)\end{aligned}\right\}$$

On p.2 of 1/28 we convinced ourselves of these limits. On Exercise 4 of 1/29 an ε-δ proof is suggested, via the geometry estimate $0 \le (1-\cos t)^2 + \sin^2 t \le t^2$

and limit theorems to check

$$\lim_{x\to c} \cos x = \cos c$$

$$\lim_{x\to c} \sin x = \sin c$$

$$\lim_{x\to c} \tan x = \tan c \qquad (\text{as long as } \cos c \neq 0)$$

you might need trig identities too!

<u>Definition</u> $f(x)$ is <u>continuous</u> at $x=c$ if and only if $\lim_{x\to c} f(x) = f(c)$

(If $f(x)$ is continuous at $x=c$ you can compute $\lim_{x\to c} f(x)$ by "plugging in" $f(c)$.)

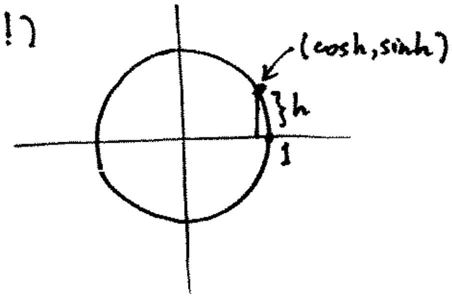
So far, we have shown polynomials are continuous at every $x=c$, also $\sin x$, $\cos x$, also rational functions at pts where the denom $\neq 0$.

Important trig limits: (see webworks too!)

(a) $\lim\limits_{h\to 0} \dfrac{\sin h}{h} = 1$

(b) $\lim\limits_{h\to 0} \dfrac{1-\cos h}{h^2} = \dfrac{1}{2}$

(c) $\lim\limits_{h\to 0} \dfrac{1-\cos h}{h} = 0$

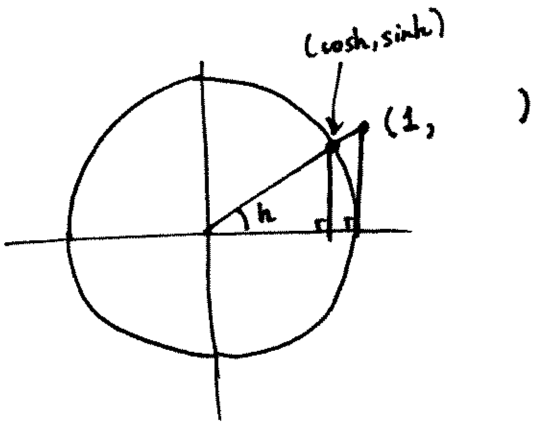


<u>Exercise 2</u>: Notice than $\dfrac{\sin h}{h}$ is even, since $\dfrac{\sin(-h)}{-h} = \dfrac{-\sin h}{-h} = \dfrac{\sin h}{h}$.

Therefore, to see (a), we need only check

$$\lim_{h\to 0^+} \frac{\sin h}{h} = 1.$$

Use the following diagrams and area formulas, and the <u>squeeze theorem</u> to prove this fact.



if $f(x) \le g(x) \le h(x)$ on an open interval containing c, and if $\lim\limits_{x\to c} f(x) = L$ and $\lim\limits_{x\to c} h(x) = L$

then $\lim\limits_{x\to c} g(x) = L$

<u>Exercise 3</u>: Use the double angle formula for cos, in the form $\cos\left(2\frac{h}{2}\right) = 1 - 2\sin^2 h/2$ to prove (b)

<u>Exercise 4</u> Deduce (c) from (b)

Limit (a) leads to lots of "fun" limits, see e.g. §1.4 problems, and webworks #4.

<u>Exercise 5</u> (#8 §1.4)

$$\lim_{\theta \to 0} \frac{\tan 5\theta}{\sin 2\theta}$$

(#11 §1.4)

$$\lim_{t \to 0} \frac{\tan^2 3t}{2t}$$

<u>Exercise 6</u> :

$$(\cos x)' = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$(\sin x)' = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

Compute these limits! (see webworks too!)

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Exam 1 Spring 2008

n = 99