Math 1210-3
Tuesday 2/26

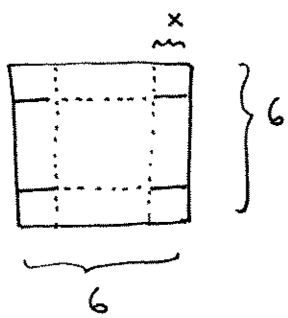
Wednesday night exam review session
7–8:30 p.m. JFB 102
NOT HERE!

§3.1 <u>Maxima and minima</u> (also introduce all of 3.1–3.5!)

In science, mathematics, business one seeks (or nature seeks) to optimize an outcome or output — mathematically this means that there are functions which are being maximized or minimized

Warmup exercise (for the theory described in §3.1):

<u>Exercise 1</u> : Your kids want to make baskets of paper squares which are 6 in by 6 in. They will cut and fold according to this diagram:



Find x so that the resulting open-top basket has the most volume possible.
Follow these steps:

1a) Volume $V(x)=$
1b) Physically reasonable domain?
1c) If the maximum value of V occurs at some x inside the domain (i.e. not one of the endpoints), then we expect $V'(x)=0$ there. Why?

1d) Find the dimensions of the open-top box with the largest volume. - using logic & 1c).

1e) (this highlights later sections in the text)
- sketch the graph of $y=V(x)$ for its natural domain of $-\infty<x<\infty$. Include 1st & 2nd derivative information. Verify that this more complete description of $y=V(x)$ is consistent with 1d).

ideas in § 3.2–3.5!

Plot of $y = 4x(x-3)^2$
$= 4x^3 - 24x^2 + 36$

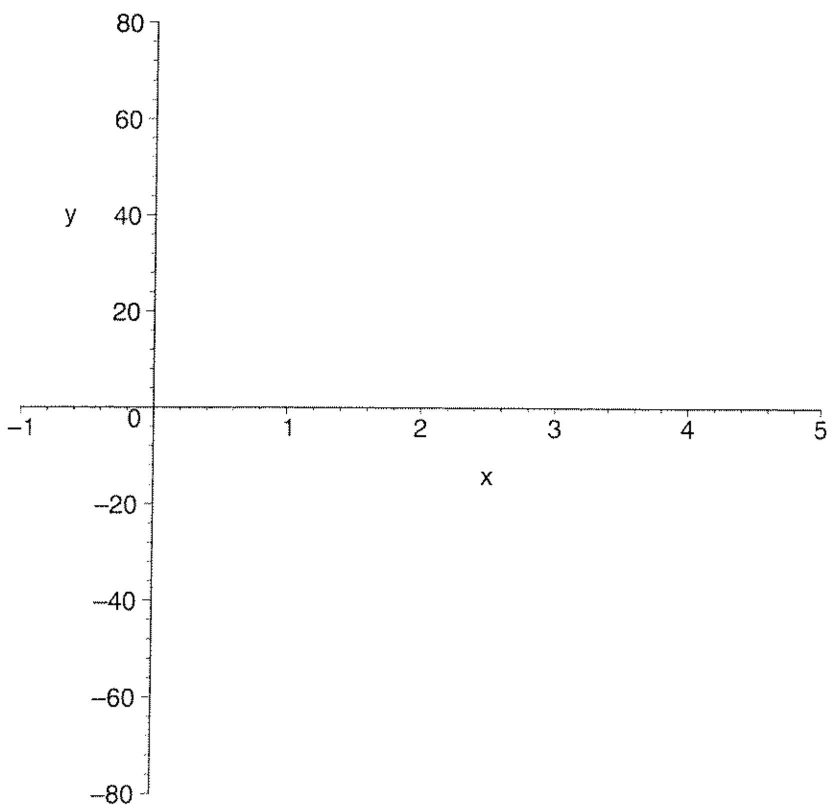
y': information about where the graph slope is positive, negative, zero

y'': if $(\text{slope})' < 0$:
if $(\text{slope})' > 0$:

concave down CD

concave up CU

x	y



Babk to §3.1...
We will use some believable facts (which are proven in more advanced math classes) to organize our work in "max-min" problems.

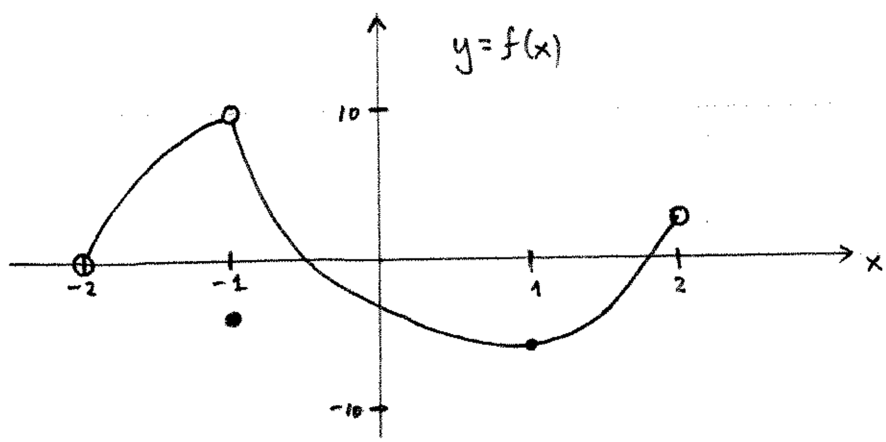
First, definitions:

Let S be the domain of a function f (typically S is an interval)
Let c be a point in S
Then

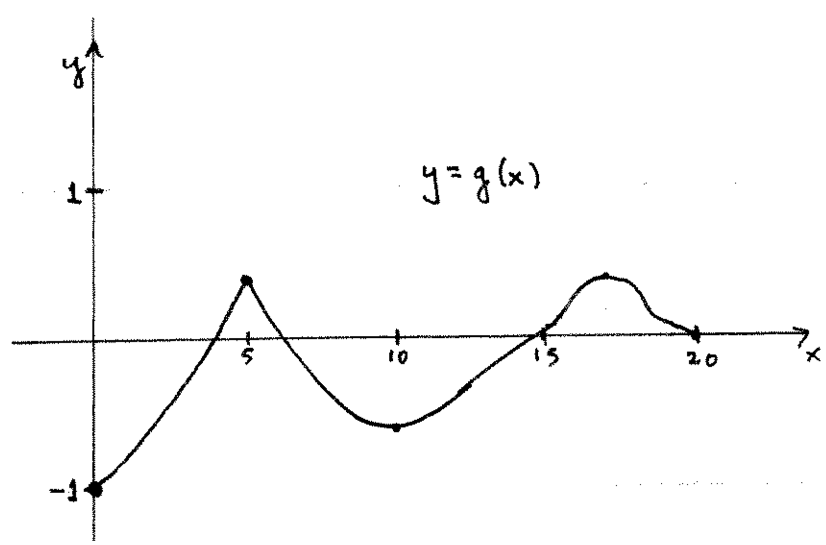
(i) $f(c)$ is a <u>maximum value</u> of f on S, if $f(c) \geq f(x)$ for all x in S
(ii) $f(c)$ is a <u>minimum value</u> of f on S, if $f(c) \leq f(x)$ for all x in S
(iii) $f(c)$ is an <u>extreme value</u> of f on S if it is either a maximum or a minimum value.

<u>Exercise 2</u> From the graphs of functions shown below, and the corresponding domains, find all extreme values, and the points at which they occur.

2a)
$S = (-2, 2)$
i.e. $-2 < x < 2$
an open interval



2b) $S = [0, 20]$
i.e. $0 \leq x \leq 20$
a closed interval
and a continuous function



<u>Theorem A</u> If f is continuous on a closed interval $S = [a, b]$, then f attains a maximum and a minimum value on that domain. (e.g. 2b)

Theorem B (says where extreme values can occur)

Let f be defined on an interval I. If $f(c)$ is an extreme value, then either

(i) c is an endpoint of I

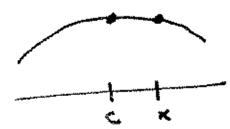
(ii) $f'(c)$ exists an equals 0 (In this case c is called a stationary point)

(iii) $f'(c)$ does not exist (In this case c is called a singular point

proof: Consider the case $f(c)$ is a maximum value.

If c is not an endpoint, and $f'(c)$ does exist, that means we are not in cases (i), (iii).

So let's show $f'(c)=0$:



For $x > c$, $f(x) \le f(c)$ so $f(x)-f(c) \le 0$

so $\dfrac{f(x)-f(c)}{x-c} \le 0$

so $\displaystyle\lim_{x\to c^+} \frac{f(x)-f(c)}{x-c} \le 0$

i.e. $\boxed{f'(c) \le 0}$

For $x < c$, $f(x) \le f(c)$ so $f(x)-f(c) \le 0$

so $\dfrac{f(x)-f(c)}{x-c} \ge 0$ (since $x-c<0$)

so $\displaystyle\lim_{x\to c^-} \frac{f(x)-f(c)}{x-c} \ge 0$

i.e. $\boxed{f'(c) \ge 0}$ thus $f'(c)=0$!

We call (i) endpoints

(ii) stationary points

or (iii) singular points

critical points.

By Theorem A, & Theorem B if f is continuous on $[a,b]$, we can find the maximum and minimum values (and where they occur), by checking f at all critical points!

Exercise 3 Label all the critical points in exercises 2a), 2b).