Math 1210-3
Monday Feb 25

First, finish related rates problems from last week.
Then,

§2.9 Differentials and approximation

Recall, if $y = f(x)$

$f'(x)$ and $\frac{dy}{dx}$ are two notations for the derivative.

If we write

$$\frac{dy}{dx} = f'(x)$$

and think of $\frac{dy}{dx}$ as a quotient, with numerator dy and denominator dx,

this motivates the definition

$$\boxed{dy := f'(x)\,dx}$$

"dx" and "dy" are called <u>differentials</u>.

<u>Exercise 1</u>

1a) If $y = \sqrt{x^2+3x}$, express dy in terms of dx

1b) If $A = \pi r^2$, express dA in terms of dr

1c) If $V = \frac{\pi}{12} h^3$, express dV in terms of dh

What good are differentials?

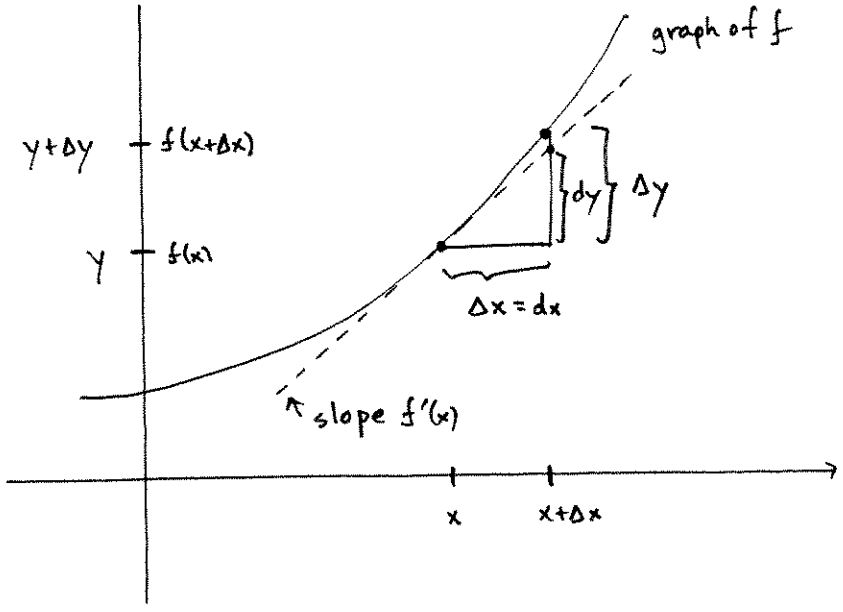
Ans: In §3.8, 4.4 we'll see how differentials guide us in using the chain rule backwards, for antidifferentiation & integration.

In the current section we see how differentials can be used for approximation and error analysis.

Here is the picture which explains how differentials allow one to approximate function values $f(x+\Delta x)$, in terms of $f'(x)$ and Δx. (Here you think of x as fixed, and Δx as varying)

Set $dx = \Delta x$ $\qquad \Delta y = f(x+\Delta x) - f(x)$ (exact change in f)

$dy = f'(x)\,dx$, $\qquad dy$ is an approximation to Δy, using the tangent line at $(x, f(x))$ to approximate the actual graph



$dy \approx f(x+\Delta x) - f(x)$,

so $\boxed{f(x+\Delta x) \approx f(x) + dy}$

Exercise 2 Use $y = \sqrt{x}$ and differentials, approximate

2a) $\sqrt{4.2}$ $\quad (x = 4,\ dx = .2)$ $\qquad$ exact val: 2.04939...

2b) $\sqrt{8.7}$ $\qquad$ exact val 2.94958...

Exercise 3 Use differentials to estimate the volume of paint necessary to paint a hemisphere of radius 5 feet, with a layer of paint .05 inches thick.

Answer in gallons ?!

$1\ ft^3 = 12^3\ in^3$

$7.481\ gal = 1\ ft^3$

Compare to "exact"

Exercise 4 The side lengths of a cube are measured to be $10\ cm \pm .1\ cm$. Use differentials to estimate how close the volume is to $10^3\ cm^3$.

Compare to "exact" error est.