Math 1210-3
Wednesday Feb. 13

§2.5-2.6 Chain rule
higher order derivatives

Finish webworks 4 !!!
Friday quiz 2.1-2.5

Exercise 1

$(fg)' =$

$(fgh)' =$

$(g^n)' =$

$(f/g)' =$

$(f\circ g)'(x) =$

$(f\circ g\circ h)'(x) =$

Practice the chain rule !!

Exercise 2 (left-overs from yesterday + new examples)

Write each function as $f\circ g$, and use chain rule to differentiate:

2 a) $F(r) = (\tan r)^4$, find $D_r F(r)$

2b) $G(t) = [\cos(3t^3)]^{10}$, find $D_t G(t)$

(do you see why we call it the chain rule?)

2c) $F(\theta) = (1+\sec^2\theta)^{-5}$, find $D_\theta F(\theta)$.

Derivative notations, and higher order derivatives :

For $y = f(x)$

The derivative $\left(\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \right)$ has notations

$\boxed{\text{a) } f'(x) \text{ or } y'}$

$\boxed{\text{b) } D_x f(x) \text{ or } D_x y}$

There is a 3^{rd} type of derivative notation, due to Wilhelm Leibniz (who co-discovered Calculus with Newton)

from yesterday, recall that for $y = f(x)$, and Δx $\Delta y = f(x+\Delta x) - f(x)$

so $f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$, leads to

$\boxed{\text{c) } \frac{dy}{dx} \text{ or } \frac{d}{dx} f(x)}$ Leibniz notation.

the chain rule looks neat in Leibniz notation

if $y = f(u)$
& $u = g(x)$

then $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$ $\xrightarrow{\text{same as}}$ $D_x f(g(x)) = f'(g(x))\, g'(x)$

(think of the du's cancelling)

and the Leibniz notation mirrors the explanation for the chain rule that we gave yesterday:

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta u} \frac{\Delta u}{\Delta x} \quad !$$

$\frac{\Delta y}{\Delta u} \to \frac{dy}{du}$ since $\Delta u \to 0$ as $\Delta x \to 0$.

$\frac{\Delta u}{\Delta x} \to \frac{du}{dx}$

Exercise 3 Practice recognizing the different notations for derivative, and computing

3a) Find $G'(1)$ for $G(t) = (t^2+9)^3 (t^2-2)^4$

3b) Find $\frac{dy}{dx}$ for $y = \left(\frac{\sin x}{\cos 2x}\right)^3$

Higher order derivatives: the derivative of the derivative of $f(x)$ is the 2^{nd} deriv. of f. The derivative of the 2^{nd} deriv is the 3^{rd} deriv., etc.

(Example: $s(t)$ = position at time t, then $s'(t) = v(t)$, and 2^{nd} deriv of s is accel.)

Notation!! We consider $\boxed{y = f(x)}$

Derivative	prime notation	D notation	Leibniz notation	
first	$f'(x)$ or y'	$D_x f(x)$ or $D_x y$	$\frac{dy}{dx}$	
second	$f''(x)$ or y''	$D_x^2 f(x)$ or $D_x^2 y$	$\frac{d^2y}{dx^2}$	("=" $\frac{d}{dx}\frac{dy}{dx}$)
third	$f'''(x)$ or y'''	$D_x^3 f(x)$ or $D_x^3 y$	$\frac{d^3y}{dx^3}$	("=" $\frac{d}{dx}\frac{d}{dx}\frac{dy}{dx}$)
n^{th}	$f^{(n)}(x)$ or $y^{(n)}$	$D_x^n f(x)$ or $D_x^n y$	$\frac{d^ny}{dx^n}$	

Exercise 4 If $y = \sin 2t$, find

4a) y'

4b) $D_t^2 y$

4c) $\frac{d^3y}{dt^3}$

4d) $y^{(10)}(t)$