Math 1210-3
Tuesday Feb. 12

- Review derivative rules on page 1 Monday notes. (§2.3-2.4)
  (Don't forget the triple product rule, it's kind of neat.)

§2.5
- Then, tackle the chain rule for differentiating compositions $f \circ g$.

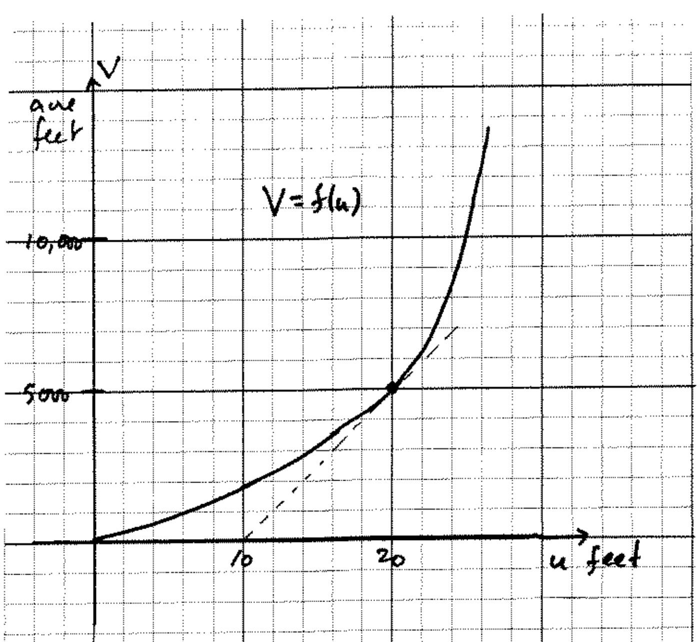
I decided to rewrite pages 2-3 of Monday's notes, to make the letters consistent on pages 2&3. (The content is the same.):

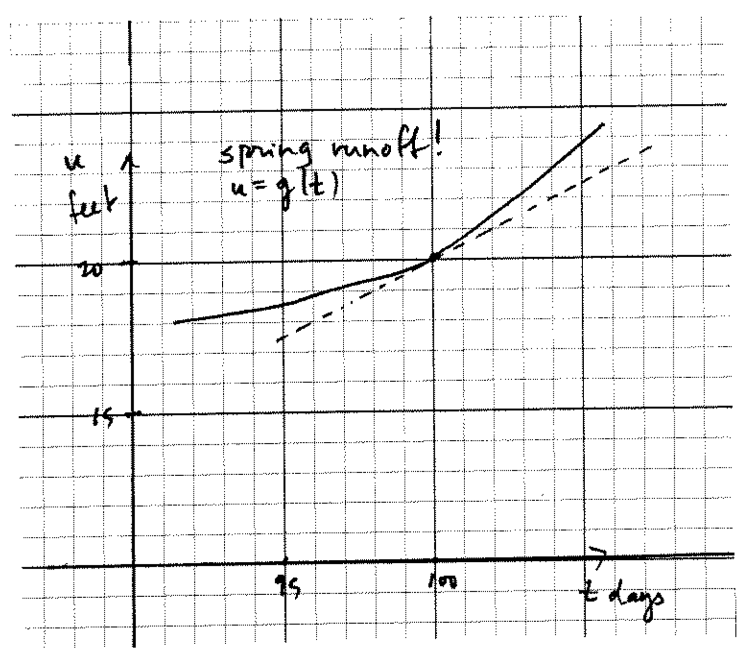
<u>Exercise 1</u> If your dog is twice as fast as you
& you're three times faster than your little brother,
<u>then</u> how many times faster is your dog than your little brother?

<u>Exercise 2</u> The volume of a town's resevoir (in <u>acre-feet</u>) depends on the maximum water depth u, $\boxed{V = f(u)}$. The depth at day t of a given calendar year is a function of t, $\boxed{u = g(t)}$

u ↑ measured in feet

Suppose at $t = 100$ (March something) we have
$u = 20$ feet
$V = 5000$ acre feet.
Consult the two graphs below to answer questions on the next page





Exercise 2a) What's happened to the water depth between $t=100$, $t=101$?
(Use tangent line to approximate)

Exercise 2b) What's happened to resevoir volume between $t=100$, $t=101$?
(Use tangent line to approximate)

Exercise 2c) About how fast is resevoir volume changing at $t=100$?
use correct units.

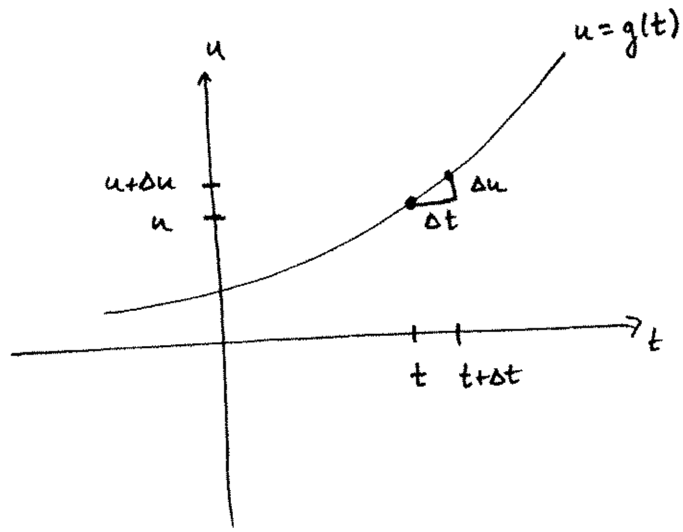
<u>Moral</u>: rates multiply.

Let's discuss $D_t(f(g(t))$ carefully, using (new) notation that typically gets introduced to help the discussion.

For $u=g(t)$ we write "Δt" for "change in t", instead of "h"
and "Δu" for "change in u", i.e. $\Delta u = g(t+\Delta t)-g(t)$.

So, converting notations,

$$D_t\, g(t) = \lim_{h\to 0} \underbrace{\frac{g(t+h)-g(t)}{h}}_{\uparrow \text{ original notation}} = \lim_{\Delta t \to 0} \underbrace{\frac{\Delta u}{\Delta t}}_{\uparrow \text{ new notation}}.$$



Use these notation conventions, and consider the composition

$$u = g(t) \qquad g'(t) = \lim_{\Delta t \to 0} \frac{\Delta u}{\Delta t}$$

$$V = f(u) \qquad f'(u) = \lim_{\Delta u \to 0} \frac{\Delta V}{\Delta u}$$

Then

$$D_t f(g(t)) = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta u} \cdot \frac{\Delta u}{\Delta t} \qquad \text{(assuming } \Delta u \neq 0\text{)}$$

$$= f'(u)\, g'(t)$$

$$\boxed{D_t f(g(t)) = f'(g(t))\, g'(t)}$$

this is the chain rule.
the letter "t" could be replaced with "x" (or any other letter.)

Luckily, once you get the hang of it, the chain rule is not so bad.
But, it <u>takes practice</u>!

<u>Exercise 3</u> Write each composition F as $f \circ g$, and use the chain rule to find the derivs.

3a) $F(x) = (x^3 + x)^2$. (Check with "foil")

3b) $F(t) = \sin(5t^2)$

3c) $F(r) = (\tan r)^4$

3d) $G(t) = [\cos(3t^3)]^2$