Name Solutions

Student I.D. ____________________

Math 1210-3
Exam #2 retake problems 4,5
March 7, 2008

Please show all work for full credit. This exam retake is closed book and closed note, but you are allowed a scientific calculator. (NOT a graphing calculator, though.) There are 25 points possible on the related rates problem and 10 possible on the differentials problem. You may choose to work one or both problems. If either score is higher than it was on your original exam, we will replace your original score(s) with the improved one(s).

5) The square root of 16 is 4. Use differentials to estimate the square root of 15.5. (As a check, you may wish to compare your approximation to your calculator's decimal value for the square root of 15.5.)
(10 points)

$\sqrt{16} = 4$

$\sqrt{15.5} = ?$

$y = x^{1/2}$ @ $x = 16,\ dx = -.5$

$dy = \frac{1}{2} x^{-1/2} dx$

$= \frac{1}{2} \frac{1}{\sqrt{16}} \left(-\frac{1}{2}\right)$

$= \frac{1}{2} \cdot \frac{1}{4} \left(-\frac{1}{2}\right)$

$= -\frac{1}{16}$

$\sqrt{15.5} = y + \Delta y$

$\approx y + dy$

$= 4 - \frac{1}{16}$

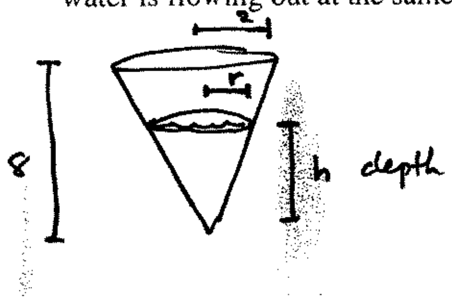
$= 3\frac{15}{16}$

≈ 3.9375

vs. calculator value: $\sqrt{15.5} \approx 3.9370$

4a) A mountain cabin has a drinking-water cistern, shaped like an upside-down cone. The depth is 8 feet, and the circular top has a radius of 2 ~~feet. The cistern is filled with water from a spring. After being totally emptied, it is being refilled.~~ When the depth of the water is 4 feet, the depth is increasing at a rate of 3 inches per minute. ~~How~~ fast is water flowing into the cistern at that instant, assuming no ~~water is flowing~~ out at the same time?

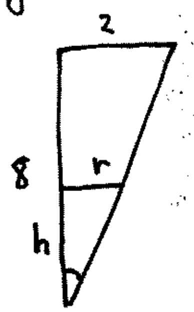
(20 points)



When $h = 4'$, $h'(t) = 3''/\text{min} = \frac{1}{4}$ ft/min

Find $V'(t)$ at that inst, V = volume.

only need 2-d:



$V = \frac{1}{3}\pi r^2 h$

similar Δ's : $\frac{r}{h} = \frac{2}{8} = \frac{1}{4}$ so $r = \frac{1}{4}h$

so $V = \frac{1}{3}\pi\left(\frac{1}{4}h\right)^2 h$

$V = \frac{\pi}{48}h^3$

D_t: $V' = \frac{\pi}{48}\, 3h^2 h'$

@ instant, $V' = \frac{\pi}{48} \cdot \cancel{3} \cdot \cancel{4^2} \cdot \frac{1}{4} = \boxed{\frac{\pi}{4} \text{ ft}^3/\text{min}}$

$\cancel{16 \cdot 3}$

4b) Assuming the inflow rate remains constant and that no one is using water from the cistern, how much later will the cistern be completely refilled?

(5 points)

currently $V = \frac{\pi}{48} \cdot 4^3$

need to fill to $\frac{\pi}{48} \cdot 8^3$

difference is $\frac{\pi}{48}(8^3 - 4^3)$ ft^3 at rate of $\frac{\pi}{4}$ ft^3/min, takes

$$\boxed{\frac{\frac{\pi}{48}(8^3-4^3)}{\pi/4} \qquad \frac{\text{ft}^3}{\text{ft}^3/\text{min}} = \text{min}}$$

$= \frac{1}{12}(8^3 - 4^3) = \frac{1}{3}(2 \cdot 64 - 16) = \frac{128-16}{3} = \frac{112}{3} = \boxed{37\frac{1}{3} \text{ minutes}}$