Math 1210-3
Tuesday April 8

§4.5 Integral mean value theorem (continued),
and
symmetry simplifications in integration.

recall, the average value of f on $[a,b]$ is defined as

$$f_{av} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

and the mean value theorem for integrals says that for f continuous on $[a,b]$ there is at least one c, $a<c<b$, with $f(c) = f_{av}$.

<u>Exercise 1</u> For $f(x) = 3 + \frac{1}{2}x$, $0 \le x \le 4$,
find f_{av}.

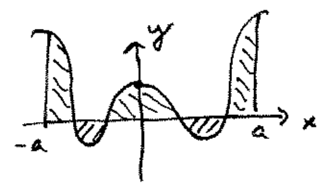
find c in $(0,4)$ with $f(c) = f_{av}$.
sketch.

<u>Exercise 2</u> For $f(x) = \sin x$, $0 \le x \le \pi$
find f_{av}.

how many c's, $0<c<\pi$,
have the property that $f(c) = f_{av}$?
what are the c-values?
sketch.

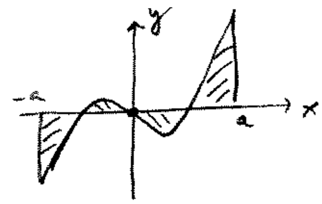
Symmetry shortcuts in definite integrals

(1) If $f(x)$ is **even**, $\quad \int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$
$(f(-x) = f(x))$



check by substitution:

(2) If $f(x)$ is **odd**, $\quad \int_{-a}^{a} f(x)\,dx = 0$
$(f(-x) = -f(x))$

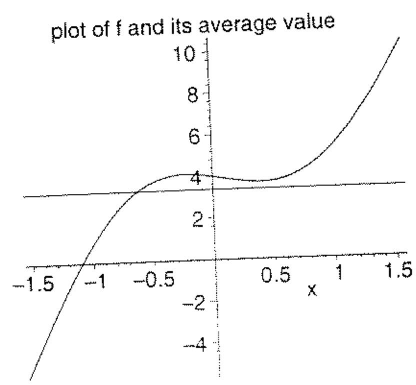


check:

Exercise 3: Use symmetry to compute

a) $$\int_{-\pi/2}^{\pi/2} x^3 + x^2 + x^2\sin x - x\cos(2x) + 4\cos x \, dx$$

b) The average value of $f(x)$, the integrand in (a), for $-\pi/2 \le x \le \pi/2$.



3.37
you **don't** want to use:

$$\int x^2 \sin(x)\,dx = -x^2\cos(x) + 2\cos(x) + 2x\sin(x) + C$$

$$\int x\cos(2x)\,dx = \frac{1}{4}\cos(2x) + \frac{1}{2}x\sin(2x) + C$$

there is a 3rd symmetry shortcut – this one is for periodic functions:

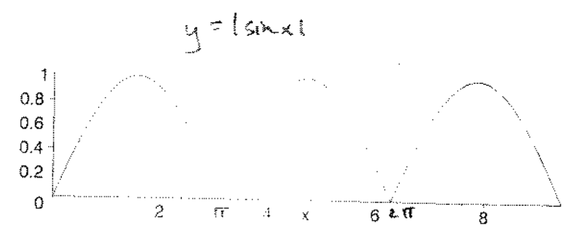
(3) If f is periodic with period p (i.e. $f(x+p) = f(x)$ for all x)
then

3a) $$\int_{a+p}^{b+p} f(x)\,dx = \int_a^b f(x)\,dx$$

3b) $$\int_a^{a+p} f(x)\,dx = \int_0^p f(x)\,dx$$ (if you integrate f over one period, it doesn't matter where you start)

check!

for 3b), first consider the case $0 \le a < p$,
then use 3a) for the general case.

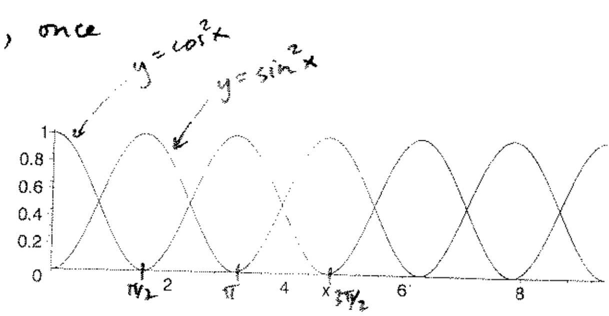


Exercise 4 Compute

a) $$\int_0^{2\pi} |\sin x|\,dx$$

b) $$\int_0^{100\pi} |\sin x|\,dx$$

c) f_{av} for $f(x) = \cos^2 x$, $0 \le x \le \pi$ two ways, once by brute force, once more cleverly.



d) What is $$\int_{\pi/4}^{5\pi/4} \cos^2 x\,dx\,?$$

(use back of this sheet!)