Math 1210-3 ①
Monday April 21
First – do surface areas of revolution (Friday)

Then...
§5.5 Work

- Work done to move an object distance d against an opposing force F is

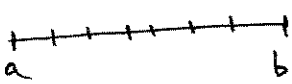
$$W = F d$$

when F is constant

$\quad\quad\quad\quad$ force dist
$\quad\quad\quad\quad\downarrow\ \ \downarrow$

<u>example</u>: To lift an object a height h on earth takes $(mg)h = W$ work
(If h is negative so is the amount of work)

- Thus, if $F(x)$, $a \le x \le b$ is a variable force



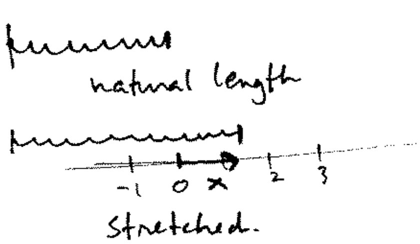
$W \approx \sum_{i=1}^{n} F(\bar{x}_i)\Delta x_i$ is approximate work to move object from a to b

$W = \int_a^b F(x)dx$ <u>is</u> the work

<u>Exercise 1</u> (p. 302). Hooke's Law (really just tangent line approx!), Says that to keep a spring stretched x units from its natural length you must exert a force proportional to x, i.e.

$$F(x) = kx.$$

(The spring's opposing force is $-kx$)



So, If a spring has natural length = .2 m
and, 12 N force is required to stretch additional .03 m (not text #)

(a) Find the spring constant k (units?)
(b) How much work is required to stretch spring from .2 m to .3 m?

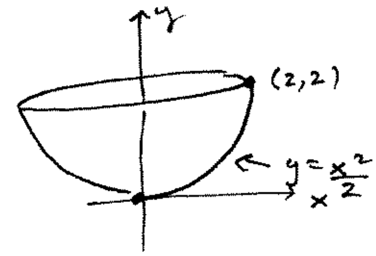
A different application of definite integrals to Work:

Exercise 2 (see p 303 for a related, but different example.)

A parabolic bowl, in the shape of a surface of revolution ($y=\frac{1}{2}x^2$, $0 \le x \le 2$ feet,) about y-axis

is full of water

(which weighs 62.4 lbs/ft^3)



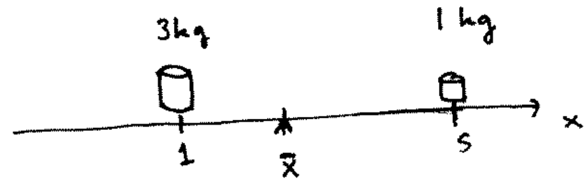
a) treating the bowl as weightless, how much work would it take to lift the entire bowl of water by **3** feet?

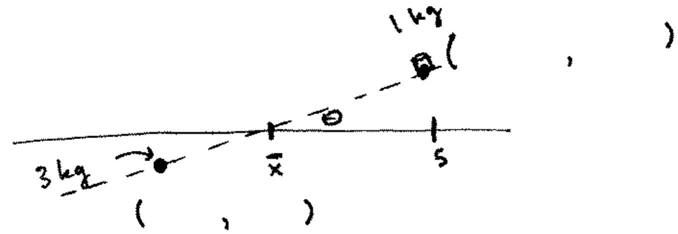
(so if the base is originally at $y=0$, it will be lifted to $y=3$).

b) How much work would it take to pump the water out of a faucet at height $y=3$, from the original bowl-water position? (This should be less work than 2a), because not all the water is lifted 3 feet!)

Exercise 3: A mass of 3 kg is placed at $x = 1$ m & a mass of 1 kg is placed at $x = 5$ m on a weightless ruler.

Find the center of mass $\bar{x}$, defined to be the balance point where it takes no net work to "teeter totter" the masses on the ruler (working against gravity):





focus on the heights, and "mgh".

In general, if you have n masses, $m_1, m_2, \ldots m_n$ located at points $x_1, x_2, \ldots x_n$

then the balance point (called the center of mass) is

$$\bar{x} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}$$

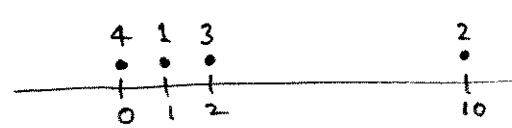
$\sum_{i=1}^{n} m_i x_i$ ← called the moment M with respect to $x=0$

$\sum_{i=1}^{n} m_i$ ← called the total mass m

Exercise 4 What is the center of mass for masses of

$m_i = 4, 1, 3, 2$ kg

at $x_i = 0, 1, 2, 10$ m ?



Tomorrow we calculusify the notion of moments and centers of mass!