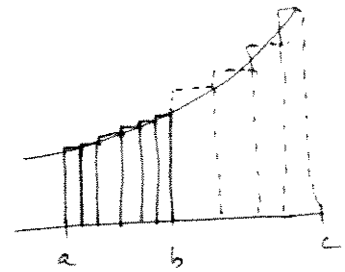
Math 1210-3
Wednesday April 2

Review sessions
tonight, JFB 102, 7–8:30 p.m.
Thurs here, 9:40–10:30 here.

3rd exam is on Friday!
Today is odds & ends + review.

- Definite integral properties
  - If $a < b < c$ then

$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$



R_P R_Q

because if R_P is a Riemann sum for $[a,b]$
& R_Q " " " $[b,c]$
then $R_P + R_Q$ is " " $[a,c]$.

and $$\int_a^c f(x)\,dx = \lim_{\substack{\|P\|\to 0 \\ \& \|Q\|\to 0}} \left(R_P + R_Q\right) = \lim_{\|P\|\to 0} R_P + \lim_{\|Q\|\to 0} R_Q$$
$$= \int_a^b f(x)\,dx + \int_b^c f(x)\,dx.$$

- Linearity of definite integral

$$\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$$

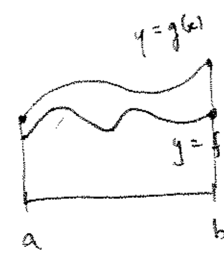
$$\int_a^b f(x)+g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)$$

because $$\sum_{i=1}^n c f(\bar{x}_i)\Delta x_i = c\sum_{i=1}^n f(\bar{x}_i)\Delta x_i$$
$$\sum_{i=1}^n \left(f(\bar{x}_i)+g(\bar{x}_i)\right)\Delta x_i = \sum_{i=1}^n f(\bar{x}_i)\Delta x_i + \sum_{i=1}^n g(\bar{x}_i)\Delta x_i$$

take the limit of these Riemann sum identities to get the definite integral identities.

- inequality estimates: If $f(x) \le g(x)$ on $[a,b]$, then

$$\int_a^b f(x)\,dx \le \int_a^b g(x)\,dx$$



Because
$$\sum_{i=1}^n f(\bar{x}_i)\Delta x_i \le \sum_{i=1}^n g(\bar{x}_i)\Delta x_i$$

Take $\lim_{\|P\|\to 0}$ $\blacksquare$

<u>Example</u> Suppose $\int_1^3 f(x)dx = 6$, $\int_1^2 f(x)dx = 2$. Find

a) $\int_2^3 f(x)dx$

b) $\int_1^3 4f(x) + 2x^2\,dx$

• Reversing limits of integration: If $a<b$, define $\int_b^a f(x)dx := -\int_a^b f(x)dx$

also $\int_a^a f(x)dx := 0.$

These definitions mean (because of FTC), that

$\int_c^d f(x)dx = F(d) - F(c)$

regardless of whether $c \le d$ or $d \le c$.

• Substitution in definite integrals

<u>Method 1</u> (done yesterday)

$$\int_a^b f(g(x))g'(x)dx \quad = \quad F(g(x))\Big]_a^b = F(g(b)) - F(g(a))$$

$u = g(x)$
$du = g'(x)dx$

$\int f(u)du = F(u) + C$

<u>Method 2</u> : Change the limits at the same time you do the u-substitution:

$$\int_a^b f(g(x))g'(x)dx$$

$u = g(x)$
$du = g'(x)dx$
$x = a \Rightarrow u = g(a)$
$x = b \Rightarrow u = g(b)$

$$\int_{g(a)}^{g(b)} f(u)du = F(u)\Big]_{u=g(a)}^{g(b)} = \text{same answer as above}$$

u-limits

<u>Example</u> Redo 1c) from yesterday, substituting limits

$$\int_0^2 x\sqrt{x^2+5}\,dx$$

Review Sheet for Exam 3

Problem session tonight JFB 102, 7–8:30 p.m.
we'll go over practice exam, which is posted.

Usual problem session tomorrow.
Exam is Friday

Exam covers 3.1–4.4

Chapter 3

Max-min problems

3.1 Extreme values and critical points
3.2 Monotonicity & concavity
3.3 local extrema, 1st & 2nd deriv tests
3.4 Practical max-min problems

- express fcn to be extremized in terms of 1 or more variables
- use constraint equations to find a function of just 1 variable to extremize, on some interval domain.
- use calculus to find extreme value.

Graphing with Calculus

3.5 (also 3.1–3.3, 1.5)

inc/dec — local max, min, neither
CU/CD — inflection pts
intercepts
asymptotes
limits related to asymptotes
sample points
plot!

Mean Value Theorem

3.6: Statement of theorem; testing theorem for given f(x) and interval [a,b]

Antiderivatives

3.8 sum rule, const. multiple rule, power rule, trig fcns, substitution using differentials (chain rule backwards).

3.9 solving DE's by antidifferentiation. separable DE's.

Newton's method

3.10 why the iteration function is $g(x) = x - \frac{f(x)}{f'(x)}$.
using Newton's method in examples.

Chapter 4.1–4.4

Definite integral and Riemann sums and FTC

4.1–4.2: Computing Riemann sums. summation notation
$\lim_{\|P\| \to 0} \sum_{i=1}^{n} f(\bar{x}_i)\Delta x$, for $\Delta x = \frac{b-a}{n}$, in special cases, to get $\int_a^b f(x)\,dx$
providing you're given the "magic formulas"
Interpreting $\int_a^b f(x)\,dx$ as signed area.

4.3–4.4 FTC 1, FTC 2. Integral properties
how to compute definite integrals via antidifferentiation
derivatives of integral expressions
applications to area

Resources

Class notes & exercises
Text
Webworks 6–8
VIP
Quizzes 7, 8
Practice test

Study advice (as always)

organize thinking by concepts
test your understanding of a concept by finding problems related to it, and making sure you can do them.