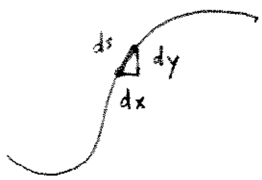
Math 1210-3
Friday April 18

§5.4 Curve lengths; Surface areas of revolution

Summary of Wednesday discussion for length of curves:



$$L = \int ds = \int \sqrt{dx^2 + dy^2} = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt \qquad \begin{matrix} x = f(t) \\ y = g(t) \end{matrix} \quad a \le t \le b$$

$$= \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \qquad y = f(x), \quad a \le x \le b$$

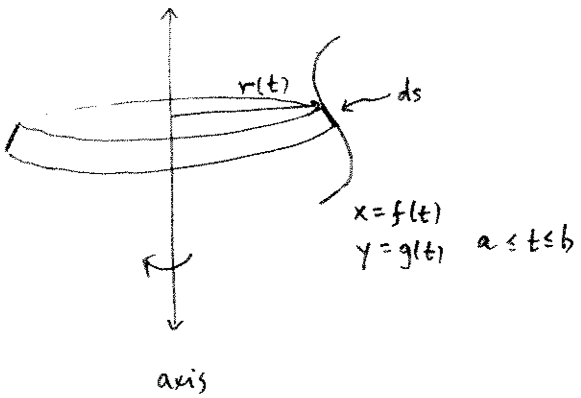
$$= \int_a^b \sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\, dy \qquad x = g(y), \quad a \le y \le b$$

Exercise 1a) $\begin{matrix} x = t^2 \\ y = 3t^2 + 4 \end{matrix}, \quad 0 \le t \le 1$

1b) $y = 3x + 4, \quad 0 \le x \le 1$
(is this related to 1a)?)

1c) $x = \frac{1}{3}y - \frac{4}{3}, \quad 4 \le y \le 7$

Surface area of revolution :



Rotate arc of length ds, get a ribbon of area

$$\Delta A \approx (2\pi \bar{r})(ds) \qquad \bar{r} = \text{average radius}$$

$$dA = 2\pi r\, ds$$

formula exact if arc is a line segment, and ribbon is a piece of a cone surface

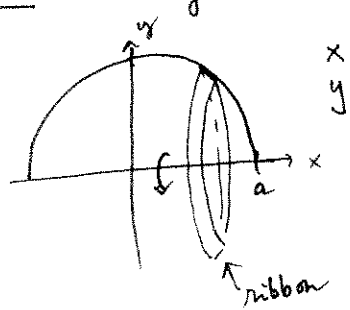
(see #31. p 301).

Leads to surface area formula, if you rotate the curve about an axis :

radius, i.e. distance from $(f(t), g(t))$ to the axis

$$A = \int_a^b \underbrace{2\pi\, r(t)}_{\text{circum}} \underbrace{\sqrt{f'(t)^2 + g'(t)^2}\, dt}_{ds}$$

Exercise 2 : Verify the formula for the surface area of a sphere



$x = a\cos t$ $\quad 0 \le t \le \pi$

$y = a\sin t$