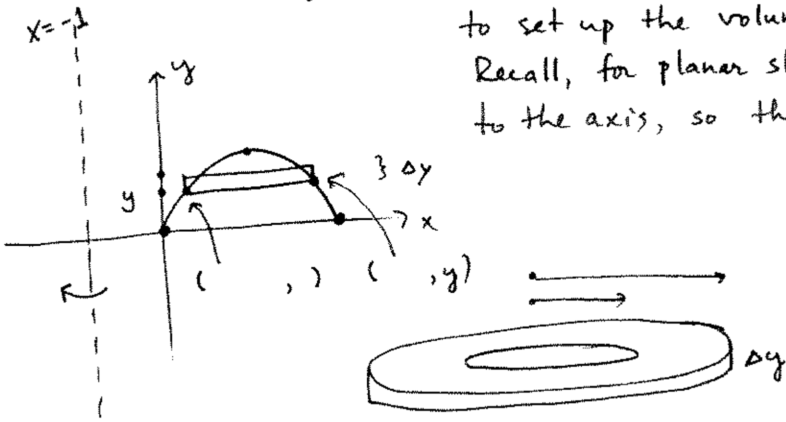
Math 1210-3
Tuesday April 15

§5.3 Volumes by cylindrical shells

Sometimes, for a volume of revolution, the slab-method (§5.2, Monday) isn't the easiest way to find the volume. ~ There's another way, "cylindrical shells"

Exercise 1 Consider the region bounded by $y = -x^2 + 2x = -(x-1)^2 + 1$ and the x-axis

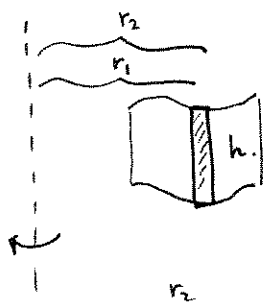
Rotate this region about the line $x = -1$. Use the planar slab method to set up the volume integral for the resulting solid. Recall, for planar slabs you always chop perpendicular to the axis, so this will be a "dy" integral.

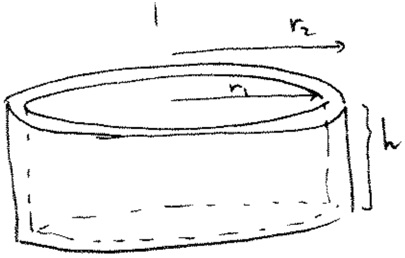


$\Delta V \approx$

$V =$

There's an easier way to compute the volume in Exercise 1, using cylindrical shells. In this case you chop the region to be rotated parallel to the axis (instead of perpendicular).



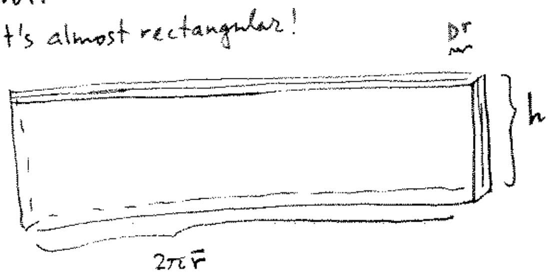


$$\Delta V = \pi(r_2^2 - r_1^2)h$$
$$= \pi\underbrace{(r_2 - r_1)}_{\Delta r}\underbrace{(r_2 + r_1)}_{2\bar{r}}h$$

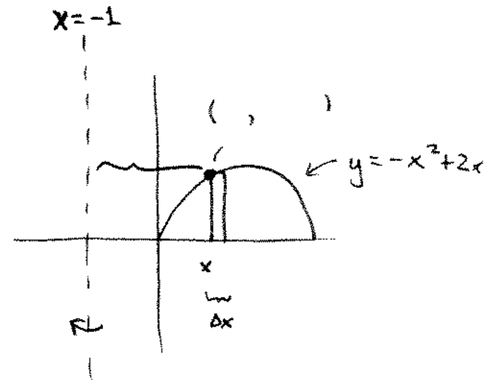
where $\bar{r} = \frac{r_1 + r_2}{2}$

$$\Delta V = 2\pi\bar{r}h\Delta r$$

Easy to remember: cut the cylinder & unroll: it's almost rectangular!



Exercise 2 Use cylindrical shells as shown at left to set up a different integral for the same volume as in Exercise 1:



$\Delta V \approx$

$V =$

Exercise 1

```
> Pi*Int((2+sqrt(1-y))^2-(2-sqrt(1-y))^2,y=0..1);
```

$$\pi \int_0^1 (2+\sqrt{1-y})^2 - (2-\sqrt{1-y})^2 \, dy$$

```
> Pi*int((2+sqrt(1-y))^2-(2-sqrt(1-y))^2,y=0..1);
```

$$\frac{16\,\pi}{3}$$

Exercise 2

```
> 2*Pi*Int((-x^2+2*x)*(x+1),x=0..2);
```

$$2\,\pi \int_0^2 (-x^2+2\,x)(x+1)\,dx$$

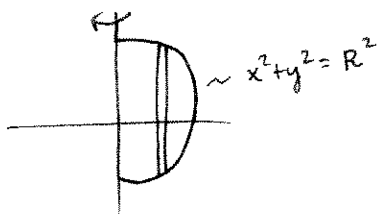
```
> 2*Pi*int((-x^2+2*x)*(x+1),x=0..2);
```

$$\frac{16\,\pi}{3}$$

```
> evalf(%);
```

$$16.75516082$$

Exercise 3 Rework the volume of a radius R ball, using cylindrical shells



Exercise 4 Rework cone volume using cylindrical shells: