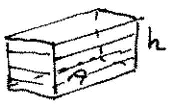
Math 1210-3
Monday April 14

§ 5.2 Volumes by planar slabs.

<u>Geometry</u>: The volume of any "slab" with cross-sectional area A and height h is

$$V = A\,h \qquad \text{(cubic length units).}$$

Because

(1) 

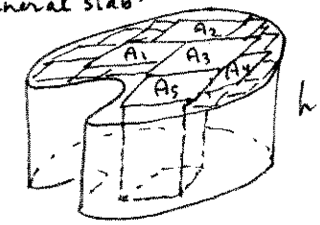
Rectangular cube

Def. of multiplication!

$V = A + A + \cdots + A$
h times (even if h is fraction)

$V = Ah$

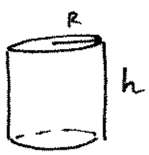
(2) General slab:



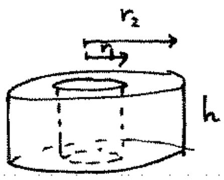
$$A = \sum_{i=1}^{\infty} A_i$$

$$V = \sum_{i=1}^{\infty} A_i h = h \sum_{i=1}^{\infty} A_i = hA$$

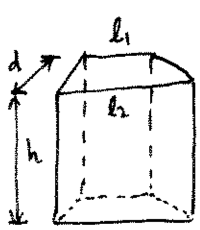
<u>Exercise 1</u> What are the volumes of the following solids:

1(a) 

solid cylinder, disk radius $= R$

1(b) 

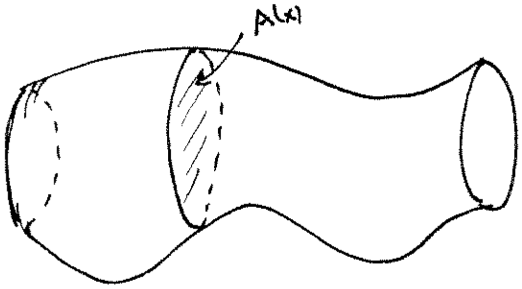
washer, with
inner radius r_1
& outer radius r_2

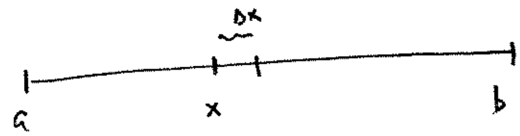
1(c) 

trapezoidal prism.

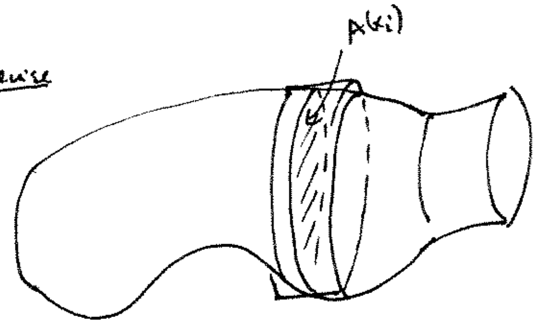
Now add Calculus! : 3-d object located between $a \le x \le b$. Continuous cross-sectional area $A(x)$

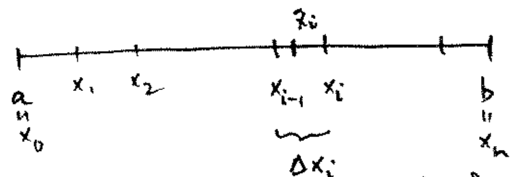
sketch:





Precise



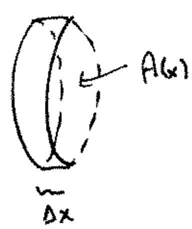


partition P & sample pts.

All we need to write down to remind us of the details on the right:

$$\Delta V \approx A(x)\Delta x$$

the area btwn x & $x+\Delta x$



$$V = \int_a^b A(x)\,dx$$

$$V \approx \sum_{i=1}^{n} A(\bar{x}_i)\Delta x_i = R_P$$

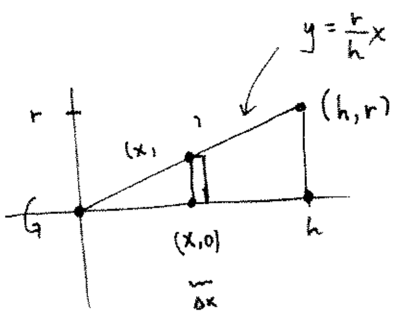
volume V defined to be

$$\lim_{\|P\|\to 0} R_P = \int_a^b A(x)\,dx$$

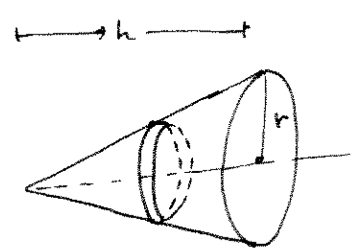
by definition of definite integral.

Exercise 2 We never did figure out why the volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

Now we can!



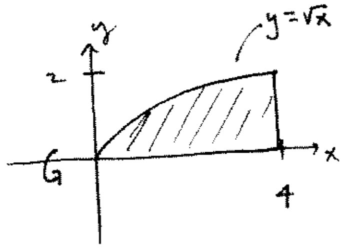
rotate triangular region about x-axis to get a solid cone of (sideways) height h, base radius r

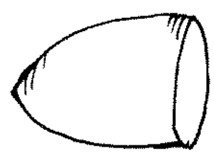


$\Delta V \approx$

$V =$

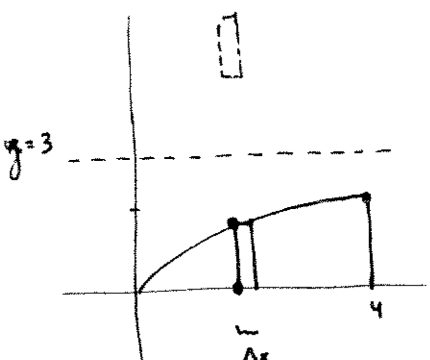
Exercise 3 Find the volume of the solid of revolution obtained by rotating the planar region bounded by $y=\sqrt{x}$, the x-axis and $x=4$ about the x-axis

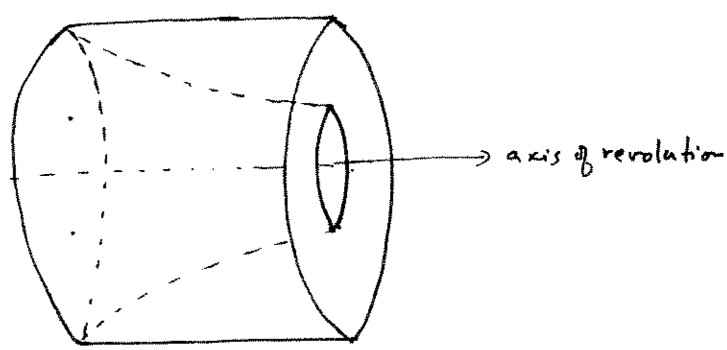




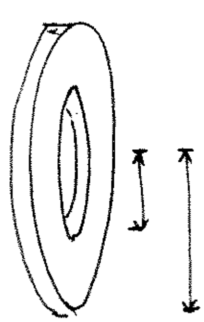
(bullet-like shape)

Exercise 4 Rotate the same region about the line $y=3$. What is the resulting solid's Volume?
Hint: Washers, not disks





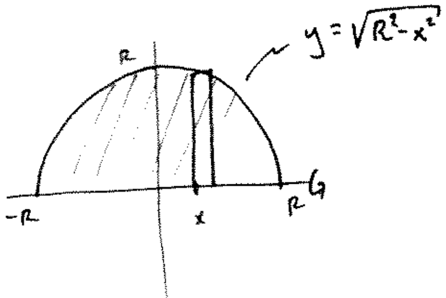
label points, deduce radii!



$\Delta V=$

$V=$

Exercise 5 Volume of ball of radius R, i.e. rotate the half-disk between $y=\sqrt{R^2-x^2}$ and x-axis about the x-axis

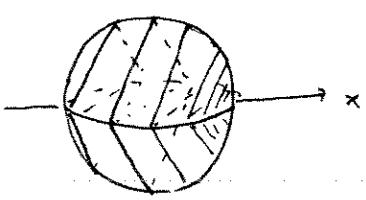


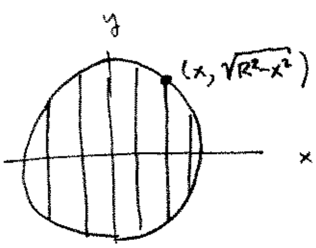
$\Delta V \approx$

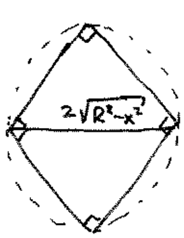
$V =$

Exercise 6 A ball of radius R is peeled, so that cross sections perpendicular to the x-axis are squares, with diagonal lengths $2\sqrt{R^2-x^2}$.

Find the volume of the peeled ball.







ans $\frac{8}{3}R^3$.