Math 1210-3
Friday April 11

5.1 : Areas of plane regions

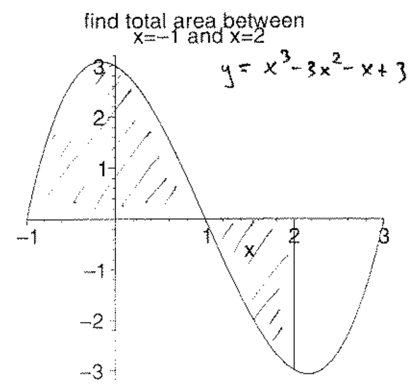
We motivated our discussion of Riemman sums and $\int_a^b f(x)dx$ by thinking about "signed area". Now that we're comfortable using FTC to compute definite integrals we can find the area of more interesting regions

Exercise 1 : Find to total area (not the signed area) between the graph of

$$y = x^3 - 3x^2 - x + 3 = (x-3)(x^2-1) = (x-3)(x+1)(x-1)$$

and the x-axis, between

$$-1 \le x \le 2$$

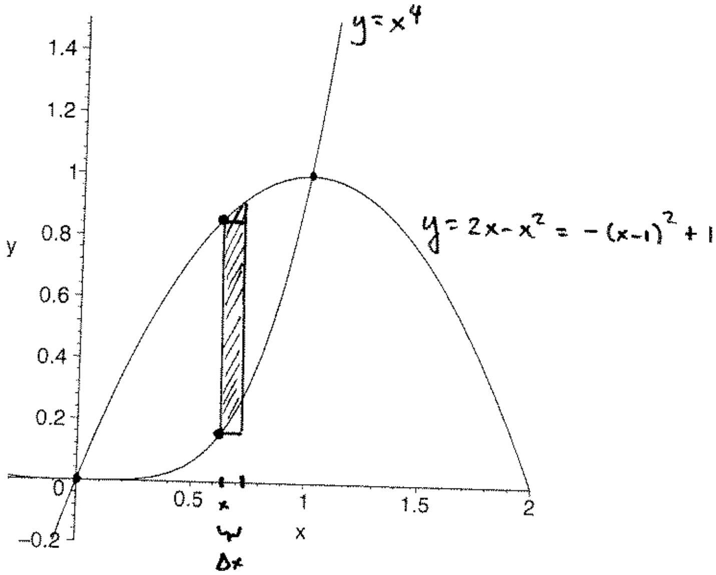


Exercise 2 Find the area of the region between $y = x^4$ and $y = 2x - x^2$.

First, find an expression which approximates the area

ΔA

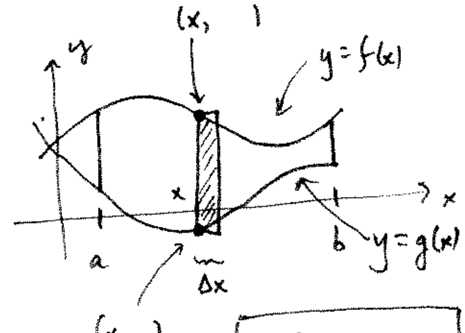
of the almost rectangular region between the two graphs, with vertical sides at x and $x + \Delta x$. Visualize how a corresponding Riemann sum would look, and write down the definite integral you'd get in the limit. Finally, compute the area.



$\Delta A \approx$

Exercise 3 Suppose $g(x) \le f(x)$ on the interval $[a, b]$.
Derive a formula for the area between the curves $y = f(x)$, $y = g(x)$, for $a \le x \le b$

Area =



$\Delta A \approx$

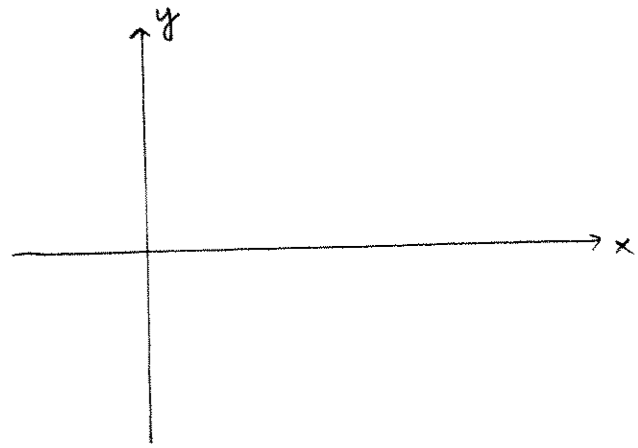
$dA =$

Sometimes it works better to slice horizontally rather than vertically!

Exercise 4 Find the area between the curves $y = x - 1$
$x = 3 - y^2$.

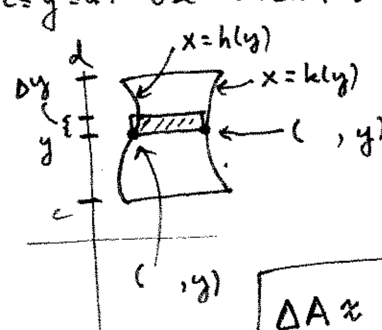
Sketch first!

4a) Use vertical slicing. You will need two definite integrals!



4b) Use horizontal slicing. It will be easier!

Exercise 5 : Suppose $h(y) \le k(y)$ for $c \le y \le d$. Find a formula for the area between the curves $x = h(y)$ and $x = k(y)$, for $c \le y \le d$. Use horizontal slicing



$A =$

$\Delta A \approx$

$dA =$