Math 1210-3
Tuesday April 1.

Problem session for Exam 3
Wednesday night 7–8:30 p.m.
JFB 102

4.3–4.4 : The First and Second Fundamental Thms of Calculus.

On Monday we used the Riemann sum definition,

$$\int_a^b f(x)\,dx := \lim_{\|P\| \to 0} \sum_{i=1}^{n} f(\bar{x}_i)\,\Delta x_i$$

to show

It was a mess !! (but we overcame it.)

a) $\int_0^3 (x+2)\,dx = 10.5$

b) $\int_{-1}^{1} (x^2+1)\,dx = 8/3$

In real science applications, sometimes one really has to resort to the definition of definite integral to approximate its value (or, use improved numerical approximations such as we will see in §4.6), because often your "$f(x)$" is just a collection of experimental data points, without a formula. But sometimes you're luckier:

Newton's (& Leibniz') conceptual breakthrough was to realize that definite integrals are related to antidifferentiation. It may (or may not) shock you to be told that the Greeks basically knew how to compute definite integrals like (b) above, essentially via the Riemann sum limit method. (It may amuse you that the English translation of what they called their technique is something like "the method of exhaustion." Bu̲t̲ the Greeks never discovered derivatives (or antiderivatives.), and so they also never discovered

THE FUNDAMENTAL THEOREM(s) OF CALCULUS.

Let $f(x)$ be continuous on $[a,b]$.

PART I (§4.3) Then $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$. That is, the "accumulation function" $G(x) = \int_a^x f(t)\,dt$ is an antiderivative of $f(x)$, so every continuous function has an antiderivative.

PART II (§4.4) Let $F(x)$ be an antiderivative of $f(x)$ on $[a,b]$.
Then $\int_a^b f(x)\,dx = F(b) - F(a)$
(write $F(x)\Big]_a^b$)

Do exercises 2,3 on Monday's notes to appreciate F.T.C. magic.

We actually talked about "why" for FTC, mar26.pdf, exercise 4,

$$\int_a^b v(t)\,dt = s(b) - s(a) \qquad \begin{array}{l} v(t) = \text{velocity} \\ s(t) = \text{position} \end{array}$$

We'll discuss the proof more carefully after the Friday midterm (when we have more time.)
For now, we'll get good at using FTC!!
The following exercises are like a lot of your webworks problems: there are some neat tricks!

Exercise 1

1a) $\displaystyle\int_{-1}^{3} 5x^2 - 3x + 1 \; dx$

1b) $\displaystyle\int_{0}^{\pi/2} 5 \sin x \; dx$

1c) $\displaystyle\int_{0}^{2} x\sqrt{x^2+5} \; dx$

1d) $\displaystyle D_x \int_{0}^{x} 1 + 2t \; dt$

1e) $\displaystyle D_x \int_{x^2}^{2x} t^2 \; dt$