V.I.P. solutions P.3 – P.5

P.3 #3 $D_x(3x-6) = 3\cdot 1 - 0 = \boxed{3}$

6) $D_x(x^{11} - 2x^9 + 15x) = 11x^{10} - 2(9x^8) + 15(1)$

$= \boxed{11x^{10} - 18x^8 + 15}$

10) $D_x(\underbrace{x^4 - 2x^3 + 5x - 3}_{f(x)}) = \underbrace{4x^3 - 6x^2 + 5}_{f'(x)}$

so at $x=2$ the slope function for $f(x)$ has value $f'(2) = 4\cdot 8 - 6\cdot 4 + 5$
$= 32 - 24 + 5$
$= 8 + 5 = \boxed{13}$

so the tangent line thru $(2, f(2)) = (2,7)$
has point-slope form
$\boxed{(y-7) = 13(x-2)}$
or slope-intercept form $y = 13x - 26 + 7$
$\boxed{y = 13x - 19}$

13) $s(t) = -16t^2 + 32t + 6$ ft
$\boxed{v(t) = s'(t) = -32t + 32}$ ft/sec
at max ht $v(t) = 0 = -32t + 32$
$32t = 32$
$\boxed{t = 1 \text{ sec}}$
max ht is $s(1) = -16 + 32 + 6 = \boxed{22 \text{ ft}}$

14) $a(t) = v'(t) = D_t(-32t + 32)$
$= \boxed{-32 \text{ ft/sec}^2}$

P.4 4) $\int 10x^9 - 8x\, dx = 10\frac{x^{10}}{10} - 8\frac{x^2}{2} + C = \boxed{x^{10} - 4x^2 + c}$

5) $\int x^2 - 5\, dx = \frac{x^3}{3} - 5x + C = F(x)$
if $F(0) = 2$ then $\frac{0^3}{3} - 5\cdot 0 + C = 2$, i.e. $\boxed{C=2}$
so $\boxed{F(x) = \frac{x^3}{3} - 5x + 2}$

9) We're on earth so $a = -32$ ft/sec² (if up is the positive direction)
so $v(t) = \int a\, dt = \int -32 dt = -32t + C$
$v(0) = 64 = 0 + C$ so $C = 64$ (in fact for these problems that "C" always equals the initial velocity, so we often write it as v_0).
so $\boxed{v(t) = -32t + 64 \text{ ft/sec}}$
max ht when $v(t) = 0$ (vel > 0 before, vel < 0 after)
$-32t + 64 = 0$
$\boxed{t = 2 \text{ sec}}$ is when ball is highest.

10) $s(t) = \int v(t)dt = \int -32t + 64 = -16t^2 + 64t + C$; $s(0) = 6 = 0 + 0 + C$, so this $C = 6$.
so $s(t) = -16t^2 + 64t + 6$.

10 cont'd $s(t) = -16t^2 + 64t + 6$

max ht is $s(2)$ (from #9);

$s(2) = -16(4) + 64(2) + 6 = -64 + 128 + 6 = 64 + 6 = \boxed{70 \text{ ft}}$ max ht.

ball lands when $s(t) = 0$.

$-16t^2 + 64t + 6 = 0$

$\div 2$: $-8t^2 + 32t + 3 = 0$

quad formula : (first mult by -1), $8t^2 - 32t - 3 = 0$

$$t = \frac{32 \pm \sqrt{32^2 - 4(8)(-3)}}{16}$$

the positive t-val is $t = \dfrac{32 + \sqrt{32^2 + 32 \cdot 3}}{16}$

$= \dfrac{32 + \sqrt{32 \cdot 35}}{16} \cong \boxed{4.09 \text{ sec}}$

P.5 1) $\displaystyle\int_1^5 x^2 - 2x + 1 \, dx = \frac{x^3}{3} - x^2 + x \Big]_1^5$

$= \frac{5^3}{3} - 5^2 + 5 - \left(\frac{1}{3} \cancel{- 1} \cancel{+ 1}\right)$

$= \frac{125}{3} - 25 + 5 - \frac{1}{3} = \frac{124}{3} - 20$

$= 41\frac{1}{3} - 20 = \boxed{21\frac{1}{3}}$ (or $\frac{64}{3}$)

$3 \overline{)124}$ $= 41\frac{1}{3}$

5) $f(x) = 3x^2 + 2x + 1$ is positive for $1 \le x \le 2$, since each term is positive,

So area $= \displaystyle\int_1^2 3x^2 + 2x + 1 \, dx = x^3 + x^2 + x \Big]_1^2 = 8 + 4 + 2 - (1 + 1 + 1) = 14 - 3 = \boxed{11}$

7) $\displaystyle\int_1^3 x^{10} - x^9 \, dx = \frac{x^{11}}{11} - \frac{x^{10}}{10} \Big]_1^3 = \frac{3^{11}}{11} - \frac{3^{10}}{10} - \left(\frac{1}{11} - \frac{1}{10}\right)$

$= 3^{10}\left[\frac{3}{11} - \frac{1}{10}\right] - \frac{10 - 11}{110}$

$= 3^{10}\left[\frac{30 - 11}{110}\right] + \frac{1}{110} = \frac{3^{10} \cdot 19 + 1}{110} \approx 10199.4$

8) $v(t) = 2t + 3t^2 + 1$

~~also~~ $s(1) = 0$.

so $s(t) = \int v(t) dt = t^2 + t^3 + t + C$

$s(1) = 0 = 1 + 1 + 1 + C$ so $C = -3$

$\boxed{s(t) = t^2 + t^3 + t - 3}$

Thus $s(3) = 3^2 + 3^3 + \cancel{3} - \cancel{3}$.

$= 9 + 27 = \boxed{36 \text{ ft}}$

(you also have worked this out using a definite integral.)