

V.I.P solutions from polynomial calculus

P.1 5) points $(-2, 3), (3, 1)$

$$\text{slope } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{3 - (-2)} = \frac{-2}{5}$$

17) thru $(1, 1)$, slope 3, so

pt-slope eqn of line is $(y-1) = 3(x-1)$

slope-intercept eqn is $y = 3x - 3 + 1$

$$y = 3x - 2$$

15) $6x - 2y = 4$

$$6x = 4 + 2y$$

$$6x - 4 = 2y$$

$$3x - 2 = y$$

$$y = 3x - 2$$

$$\begin{matrix} \text{slope} = 3 \\ y \text{ intercept} = -2 \end{matrix}$$

19) perpendicular (L)

lines have slopes which are negative reciprocals,

i.e. $m_1 m_2 = -1$

or $m_2 = -\frac{1}{m_1}$

Thus \perp line to $y = 3x + 2$ has

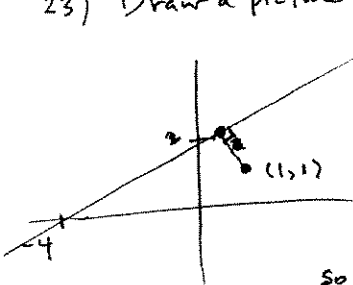
slope $m_2 = -\frac{1}{3}$, so eqn of

this \perp line is

$$(y-0) = -\frac{1}{3}(x-1)$$

$$y = -\frac{1}{3}x + \frac{1}{3}$$

23) Draw a picture to guide your work!



$$\begin{aligned} L: y &= \frac{1}{2}x + 2 \\ 2y - x &= 4 \\ 2y &= x + 4 \\ y &= \frac{1}{2}x + 2 \end{aligned}$$

L has slope $\frac{1}{2}$

so perp. line has slope -2

so pt-slope eqn is

$$(y-1) = -2(x-1)$$

$$\text{or } y = -2x + 3$$

the point of intersection

satisfies $\frac{1}{2}x + 2 = -2x + 3$

$$\frac{5}{2}x = 1$$

$$x = \frac{2}{5} = 0.4$$

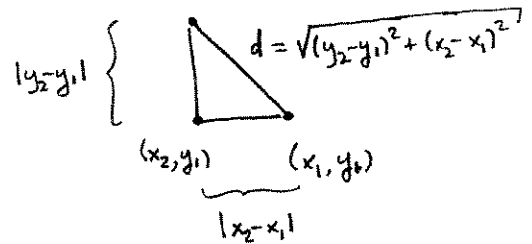
$$\text{so } y = -2\left(\frac{2}{5}\right) + 3 = 2\frac{1}{5} = 2.2$$

$$(0.4, 2.2) \text{ is intersection point}$$

The distance from this point to $(1, 1)$ is the distance from $(1, 1)$ to L. Use Pythagorean Theorem distance formula at right.

If $(x_1, y_1) = (1, 1)$
 $(x_2, y_2) = (0.4, 2.2)$ the distance $d = \sqrt{(2.2-1)^2 + (-0.4-1)^2} = \sqrt{(1.2)^2 + (-1.4)^2}$
 $= \sqrt{1.44 + 1.96} = \sqrt{3.4} \approx 1.84$

(x_2, y_2)



P.2 6a) $f(x) = 3x^2 - 2$, $x = 1$

$$\frac{f(x+h) - f(x)}{h} = \frac{f(1+h) - f(1)}{h} = \frac{[3(1+h)^2 - 2] - [3 - 2]}{h} = \frac{3(1+2h+h^2) - 2 - 1}{h} = \frac{3 + 6h + 3h^2 - 3}{h} = 6 + 3h$$

6b) $\lim_{h \rightarrow 0} 6 + 3h = \boxed{6}$

10) $f(x) = 2x + 5$

$$\frac{f(x+h) - f(x)}{h} = \frac{[2(x+h) + 5] - [2x + 5]}{h} = \frac{2x + 2h + 5 - 2x - 5}{h} = \frac{2h}{h} = 2$$

$\lim_{h \rightarrow 0} 2 = \boxed{2}$

12) $f(x) = x^2 - 2x + 3$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{[(x+h)^2 - 2(x+h) + 3] - [x^2 - 2x + 3]}{h} \\ &= \frac{x^2 + 2hx + h^2 - 2x - 2h + 3 - x^2 + 2x - 3}{h} \\ &= \frac{2hx + h^2 - 2h}{h} \\ &= 2x + h - 2 \end{aligned} \quad \lim_{h \rightarrow 0} 2x + h - 2 = \boxed{2x - 2}$$

13) $f(x) = x^3$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^3 - x^3}{h} = \frac{x^3 + 3hx^2 + 3h^2x + h^3 - x^3}{h}$$

$$\begin{aligned} (x+h)^3 &= (x+h)(x+h)^2 \\ &= (x+h)(x^2 + 2hx + h^2) \\ &= x^3 + 2hx^2 + h^2x + hx^2 + 2h^2x + h^3 \\ &= x^3 + 3hx^2 + 3h^2x + h^3 \end{aligned}$$

$\therefore \frac{d}{dx} = 3x^2 + 3hx + h^2$

$\lim_{h \rightarrow 0} (3x^2 + 3hx + h^2) = 3x^2 + 0 + 0 = \boxed{3x^2} = (x^3)'$

15) $f(x) = x^2$

$f'(x) = 2x$ is the slope function for the parabola $y = x^2$
thus when $x = -2$ the slope of the parabola is $f'(-2) = -4$.

tangent line has slope -4,
goes thru point $(-2, 4)$ so

has pt-slope eqn $\boxed{y - 4 = -4(x + 2)}$
(slope-int eqn $y = -4x - 4$)

