Name ___SOLUTIONS___

Student I.D. _____

Math 1210-4
Exam #3
April 11, 2003

Please show all work for full credit. This exam is closed book and closed note, but you may use a scientific calculator. You may not use a calculator which does graphing or symbolic differentiation, however. I have provided you with the integral tables, geometry identities and some summation formulas. There are 100 points possible, as indicated below and in the exam. Since you only have 50 minutes you should be careful not to spend too long on any one problem. Good Luck!!
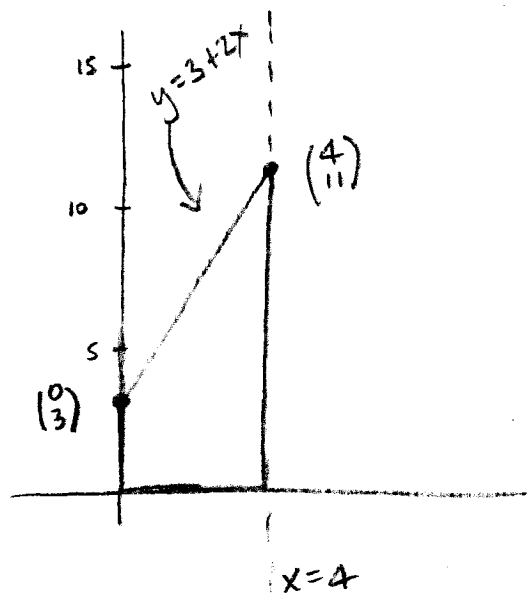
Score

POSSIBLE

1_____ 35

2_____ 30

3_____ 35

TOTAL_____ 100

1) Consider the trapezoid bounded between the graph of $y=3+2x$, the x-axis, and the lines x=0 and x=4.
1a) Sketch the region, including the coordinates of its four corners.

1b) Use geometry to calculate the area of this region.

$$A = b \cdot \tfrac{1}{2}(h_1 + h_2)$$
$$= 4 \cdot \tfrac{1}{2}(3+11) = \boxed{28}$$

1c) Use the Fundamental Theorem of Calculus to recalculate the area of this region.

$$\int_0^4 3+2x \; dx = 3x + x^2 \Big]_0^4 = 12+16 = \boxed{28}$$

1d) Now, in the last two parts of this problem, you will use the Riemann sum definition of definite integrals to recalculate the area a third time. First, divide the interval [0,4] into n equal-length subintervals. Decide how long each subinterval is, and use this information to figure out the endpoints of each subinterval. Then, using the right-hand endpoints as your sample points, write down the Riemann Sum for the function f(x)=3+2 x.

(10 points)

$$\Delta x_i = \frac{4}{n}$$
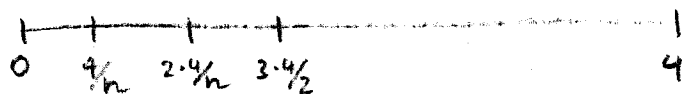
$I_1 = [0, \frac{4}{n}]$

$I_2 = [\frac{4}{n}, 2 \cdot \frac{4}{n}]$

$I_i = [(i-1)\frac{4}{n}, i\frac{4}{n}]$

$\uparrow$

$\bar{x}_i$

$f(x) = 3 + 2x$

$$R_p = \sum_{i=1}^{n} f(\bar{x}_i) \Delta x_i$$

$$\boxed{R_p = \sum_{i=1}^{n} \left(3 + \frac{8i}{n}\right)\left(\frac{4}{n}\right)}$$

1e) Express the Riemann sum in 1c) in terms of n, and then take the limit as n approaches infinity. Hopefully you will recover the area you computed in parts 1a) and 1b).

(10 points)

$$R_p = \frac{12}{n} \sum_{i=1}^{n} 1 + \frac{32}{n^2} \sum_{i=1}^{n} i$$

$$= \frac{12}{n} \cdot n + \frac{32}{n^2} \frac{n(n+1)}{2}$$

$$\boxed{R_p = 12 + 16\left(\frac{n+1}{n}\right)}$$

$$\lim_{n \to \infty} 12 + 16\left(\frac{1+\frac{1}{n}}{1}\right) = \boxed{28}$$

2) Compute the following:

2a)

$$\int_{3}^{7} 2\sqrt{5x+1}\; dx$$

(10 points)

$u = 5x+1$
$du = 5\,dx$
$\dfrac{du}{5} = dx$

$x = 3$
$u = 16$

$x = 7$
$u = 36$

$$= \int_{16}^{36} 2u^{1/2}\,\dfrac{du}{5} = \dfrac{2}{5}\,\dfrac{u^{3/2}}{3/2}\Bigg]_{16}^{36} = \dfrac{4}{15}\,u^{3/2}\Bigg]_{16}^{36}$$

$$= \dfrac{4}{15}\left(6^3 - 4^3\right) = \dfrac{608}{15}$$

$$= 40\,{}^8/_{15}$$

$\begin{array}{r} 216 \\ -\ 64 \\ \hline 152 \end{array}$

$\begin{array}{r} 40 \\ 15\,\overline{\smash{)}608} \\ \underline{60\ } \\ 8 \end{array}$

2b)

$$\int_{(-1/2)\pi}^{(1/2)\pi} [\sin(x)]^2 \cos(x)\, dx$$

(10 points)

$u = \sin x$
$du = \cos x\, du$

$x = -\pi/2$
$u = -1$

$x = \pi/2$
$u = 1$

$$= \int_{-1}^{1} u^2\, du = \dfrac{u^3}{3}\Bigg]_{-1}^{1} = \dfrac{1}{3}(1 - -1) = \dfrac{2}{3}$$

2c) The average value of $f(x) = 5 + 2\sin(x) - x^3$, on the interval $[-3, 3]$.

(10 points)

$$f_{av} = \dfrac{1}{b-a}\int_{a}^{b} f(x)\, dx$$

$$= \dfrac{1}{6}\int_{-3}^{3} 5 + 2\sin x - x^3\, dx$$

00 since interval is symmetric & these fcns are odd,

$$= \dfrac{1}{6}\cdot 30$$
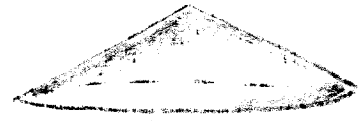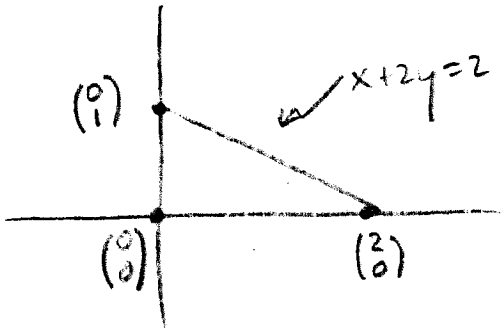
$$= 5$$

Page 4

3) Consider the triangle bounded by the x-axis, the y-axis, and the line $x + 2y = 2$. Consider the solid cone of revolution obtained by rotating the triangle about the y-axis.

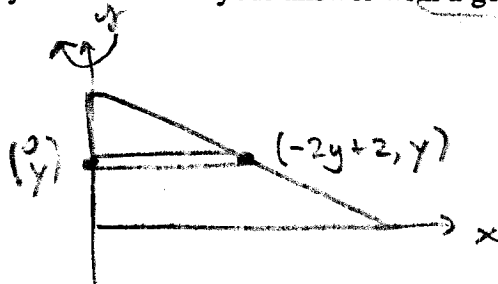3a) Sketch the triangle. Find the coordinates for the three corners.

(5 points)

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $x+2y=2$

$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$h = 1$
$r = 2$
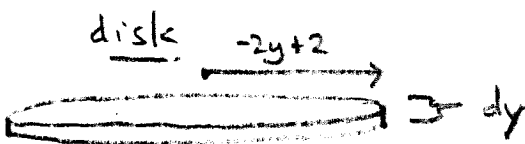$V = \frac{1}{3}\pi r^2 \cdot 1 = \boxed{\frac{4\pi}{3}}$

3c) Calculate the volume of the cone, using the method of slicing (i.e. slabs, disks or washers). You may wish to check your answer with a geometry formula.

(15 points)

$\begin{pmatrix} 0 \\ y \end{pmatrix}$ $(-2y+2, y)$

$x + 2y = 2$
$x = -2y + 2$

disk $\quad -2y+2$ $\longrightarrow$ dy
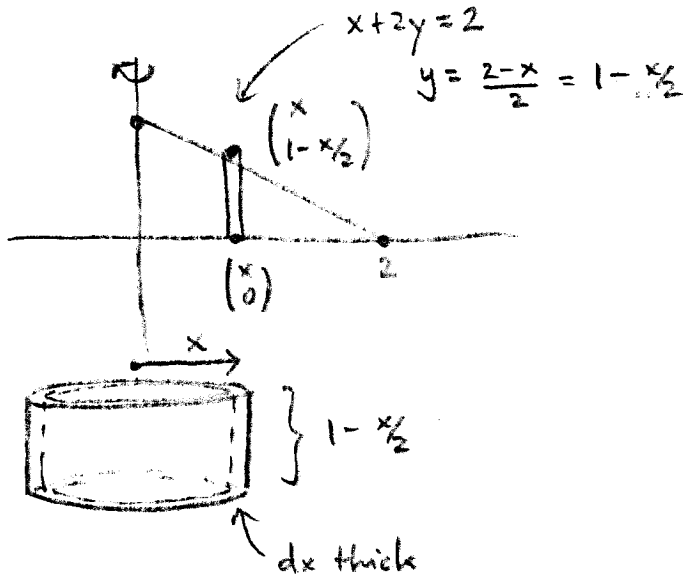
$dV = \pi(-2y+2)^2 \, dy$
$= \pi(4y^2 - 8y + 4) \, dy$

$V = \int_0^1 \pi(4y^2 - 8y + 4) \, dy = \pi\left[\frac{4}{3}y^3 - 4y^2 + 4y\right]_0^1 = \pi\left(\frac{4}{3} - 4 + 4\right)$

$= \boxed{\frac{4\pi}{3}}$ ✓

3d) Calculate the volume of the same cone, using cylindrical shells.

$$x + 2y = 2$$

$$y = \frac{2-x}{2} = 1 - \frac{x}{2}$$

$$\left(\begin{matrix} x \\ 1-\frac{x}{2} \end{matrix}\right)$$

$$\left(\begin{matrix} x \\ 0 \end{matrix}\right)$$

2

$$\Big\} 1 - \frac{x}{2}$$

dx thick

$$dV = 2\pi x \left(1 - \frac{x}{2}\right) dx$$

$$V = \int_0^2 2\pi \left(x - \frac{x^2}{2}\right) dx = 2\pi \left[\frac{x^2}{2} - \frac{x^3}{6}\right]_0^2$$

$$= 2\pi \left[2 - \frac{4}{3}\right] = 2\pi \left(\frac{2}{3}\right) = \frac{4\pi}{3}$$