Math 1210-3/4 Practice Exam 3 April 9, 2003

The same ground rules will be in effect that we've had for the first two exams. These are typical problems of the sort you may encounter - but make sure to study ALL concepts we've covered.

1) Consider the region bounded between the graph of $y=x^2$ and the x-axis, between x=0 and x=2.

1a) Use the Fundamental Theorem of Calculus to calculate the exact area of this region.

(5 points)

1b) Sketch the region. Include in this sketch the 4 rectangles one obtains by dividing the interval [0,2] into four equal-length subintervals, and using the values of $f(x)=x^2$ on the right endpoints for the height of the rectangles. Compute the Riemann sum for this partition, i.e. the sum of these rectangles' areas.

(10 points)

1c) Divide the interval [0,2] into n equal-length subintervals, and for each subinterval choose the right-hand endpoint as your sample point. Write down the resulting Riemann Sum.

(5 points)

1d) Using the summation formulas on page 223-224 (these will be provided on the exam), express the Riemann Sum in 1c), in terms of n.

(5 points)

1e) Show that the limit as n approaches infinity of your expression in 1d) equals the exact area you computed in 1a).

(5 points)

2) Compute the following:

(10 points each, total = 30)

2a)

$$\int_0^2 \frac{t^3}{\sqrt{t^4+9}}\,dt$$

2b)

$$D_x \int_0^{3x} \sin(z)^2\,dz$$

2c) Solve

$$\frac{dy}{dx}=\frac{6x-x^3}{2y}$$

with y=3 at x=0.

3) Consider the region bounded by the two curves $y=x^2$ and $y=-x+2$.

3a) Sketch the region. Find the coordinates for the intersection points of the two curves.

(5 points)

3b) Find the area of the region

(5 points)

3c) Consider the solid of revolution obtained by rotating this region about the x-axis. Find its volume.

(15 points)

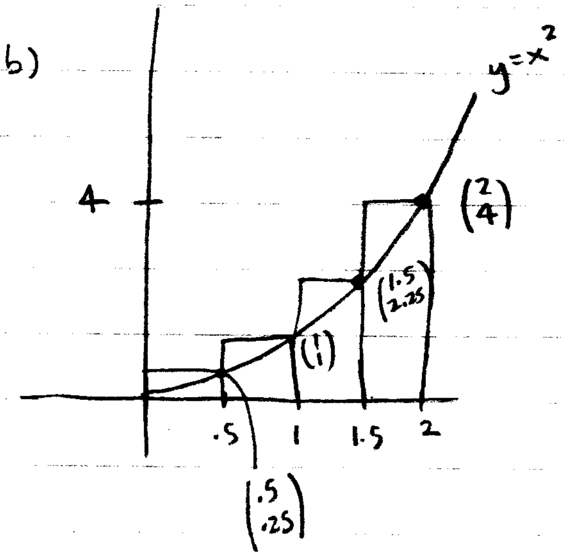
3d) Consider the solid of revolution obtained by rotating this region about the line x=2. Set up the definite integral which has value equal to its volume. You need not evaluate this integral.

(15 points)

Practice exam solutions.

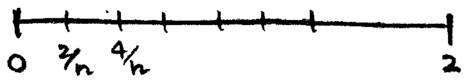
1a) $\int_0^2 x^2\,dx = \frac{x^3}{3}\Big]_0^2 = 8/3$

1b)



$R_p = .5\left[.25 + 1 + 2.25 + 4\right]$
$= .5\left[6.5\right] = 3.25$

1c)



$\Delta x_i = 2/n$

$I_i = \left[(i-1)\frac{2}{n},\ i\left(\frac{2}{n}\right)\right] \qquad i = 1, \ldots n$

$i\left(\frac{2}{n}\right) = \bar{x}_i$

$R_p = \sum_{i=1}^{n} f(\bar{x}_i)\,\Delta x_i = \sum_{i=1}^{n} \left(\frac{2i}{n}\right)^2\left(\frac{2}{n}\right)$

$f(x) = x^2$

1d) $\sum_{i=1}^{n} \frac{4i^2}{n^2}\,\frac{2}{n} = \frac{8}{n^3}\sum_{i=1}^{n} i^2 = \frac{8}{n^3}\,\frac{1}{6}\,n(n+1)(2n+1) = \frac{4}{3}\,\frac{n(n+1)(2n+1)}{n^3}$

1e) $\lim_{n\to\infty} \frac{4}{3}\left(\frac{n}{n}\right)\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right) = \lim_{n\to\infty} \frac{4}{3}(1)\left(\frac{1+1/n}{1}\right)\left(\frac{2+1/n}{1}\right) = \frac{8}{3}$

2a) $\int_0^2 \frac{t^3}{\sqrt{t^4+9}}\,dt = \int_9^{25} u^{-1/2}\,\frac{du}{4} = \frac{1}{4}\,\frac{u^{1/2}}{1/2}\Big]_9^{25} = \frac{1}{2}u^{1/2}\Big]_9^{25}$

$= \frac{1}{2}(5-3) = 1$

$u = t^4 + 9 \qquad t = 0$
$du = 4t^3\,dt \qquad u = 9$
$\qquad t = 2$
$\frac{du}{4} = t^3\,dt \qquad u = 25$

2b) $D_x \int_{f(x)}^{g(x)} h(t)\,dt = h(g(x))g'(x) - h(f(x))f'(x)$, so

$$D_x \int_0^{3x} (\sin z)^2\,dz = (\sin(3x))^2 \cdot 3$$

2c) $2y\,dy = (6x - x^3)\,dx$

$\int$: $y^2 = 3x^2 - \frac{x^4}{4} + C$

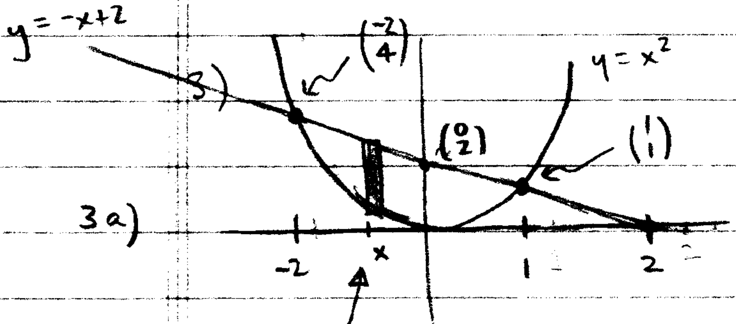
$y(0)=3$: $9 = 0 - 0 + C$; $C = 9$

$y^2 = 3x^2 - \frac{x^4}{4} + 9$

$y = \sqrt{3x^2 - \frac{x^4}{4} + 9}$

3)

3a)



(notice vertical scale is compressed relative to horizontal scale)

curves cross when

$x^2 = -x+2$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$; $x = 1, -2$

3b) $dA = [(-x+2) - x^2]\,dx$

↑ ht ↑ width

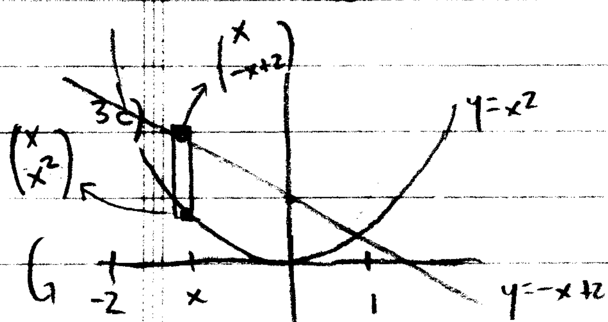
$A = \int_{-2}^{1} -x+2-x^2$

$= -\frac{x^2}{2} + 2x - \frac{x^3}{3}\Big]_{-2}^{1} = -\frac{1}{2} + 2 - \frac{1}{3}$

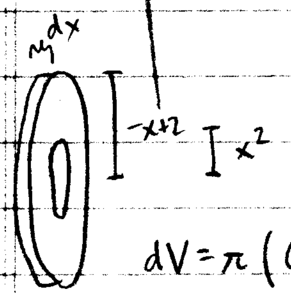
$- \left[-2 - 4 + \frac{8}{3}\right]$

$= 8 - \frac{1}{2} - 3 = 4\frac{1}{2}$

3c)



washer



$dV = \pi\left((-x+2)^2 - (x^2)^2\right)dx$

$= \pi\left(-x^4 + x^2 - 4x + 4\right)dx$

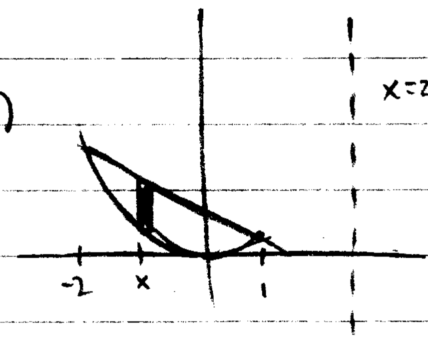
$V = \int_{-2}^{1} \pi(-x^4 + x^2 - 4x + 4)\,dx$

$= \pi\left[-\frac{x^5}{5} + \frac{x^3}{3} - 2x^2 + 4x\right]_{-2}^{1}$

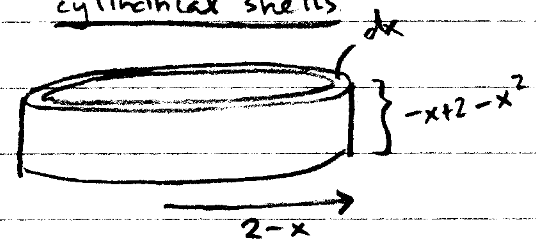
$= \pi\left[-\frac{1}{5}(1-2^5) + \frac{1}{3}(1+2^3) - 2(1-4) + 4(1--2)\right]$

$= \pi\left[\frac{31}{5} + 3 + 6 + 12\right] = \pi(27.2)$?

3d)



cylindrical shells



$dV = 2\pi(2-x)(-x+2-x^2)\,dx$

$V = \int_{-2}^{1} 2\pi(2-x)(-x+2-x^2)\,dx$

Math 1210-3/4
Wed 9 April

Review for 3rd midterm Friday

- 2nd page is practice exam. At least 80% of the exam topics are represented there. solutions will be posted tomorrow, and the Thursday problem sessions will be exam review sessions

  Thurs 9:40-10:30 OSH Auditorium
  11:50-12:40

- Exam covers §5.1-6.3

  antidifferentiation
  - polynomials
  - powers
  - trig fcns
  - substitution

  separable differential eqtns

  (substitution and separable differential eqtns: the magic of differentials)

  definite integrals with FTC
  - substitution

  FTC 1 & 2

  properties of integration (linearity, interval additivity)

  Riemann sums
  - for specific partition
  - for equi-partition of n subintervals
  - $\lim_{n\to\infty} \sum_i f(\bar{x}_i)\Delta x_i := \int_a^b f(x)\,dx$ for f cont on $[a,b]$
    - actual limit computation for equipartition
    - (summation formulas p223-224 will be provided)

  Applications
  - area
  - volumes by slicing (slabs, disks, washers)
  - volumes of revolution by cylindrical shells.
  - average value

- In addition to practice exam consider old ww's & chapter reviews p266-267
  p312-313