Math 1210-3/4
Practice Final Solutions
4/24/03

1a) $\lim_{x\to3} \frac{(\cancel{x-3})(x+2)}{(\cancel{x-3})} = \boxed{5}$

c) $\lim_{x\to\infty} \frac{\cancel{x^2}(\frac{3}{x}-1)}{\cancel{x^2}(5+\frac{2}{x}+\frac{53}{x^2})} = \boxed{-\frac{1}{5}}$

b) $\lim_{t\to1^-} \frac{t^2+1}{(t-1)(t+1)} = \frac{2}{0^-(2)} = \boxed{-\infty}$

2a) $D_x\left(24x^2+\frac{12}{x^2+1}\right) = 48x + 12(-1)(x^2+1)^{-2}2x = \boxed{48x - \frac{24x}{(x^2+1)^2}}$

b) $D_t\, t\sqrt{3t+7} = 1\cdot\sqrt{3t+7} + t\,\frac{1}{2}(3t+7)^{-1/2}3$

$\boxed{= \sqrt{3t+7} + \frac{3}{2} \frac{t}{\sqrt{3t+7}}}$

c) $D_x \frac{(\sin 2x)^5}{\cos(3x^2)} = \boxed{\frac{5(\sin 2x)^4(\cos 2x)(2)\cos(3x^2) - (\sin 2x)^5(-\sin(3x^2))6x}{(\cos 3x^2)^2}}$

d) $D_x\left(8f(x)+g(x)^3+f(g(x))\right) = 8f'(x)+3g(x)^2g'(x)+f'(g(x))g'(x)$

@ $x=1$ get $8\cdot3+3\cdot1\cdot(-2)+3\cdot(-2) = 24-6-6 = \boxed{12}$

3a) $\int 7u^3+3\sin u+2u^{-2}\,du = \boxed{\frac{7}{4}u^4-3\cos u-2u^{-1}+C}$

b) $\int_0^{\pi/2}\cos x\sin x\,dx$ $\quad u=\sin x$, $du=\cos x\,dx$; $x=0$, $x=\pi/2$; $u=0$, $u=1$ $\quad\rightsquigarrow \int_0^1 u\,du = \frac{u^2}{2}\Big]_0^1 = \boxed{\frac{1}{2}}$

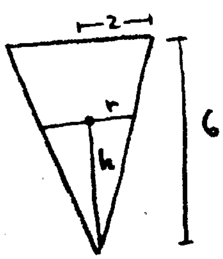
c) $\int_{-2}^3 (x^2+1)^2\,dx = \int_{-2}^3 x^4+2x^2+1\,dx = \frac{x^5}{5}+\frac{2}{3}x^3+x\Big]_{-2}^3 = \boxed{\frac{1}{5}(3^5+2^5)+\frac{2}{3}(27+8)+5}$

d) $\int\frac{t}{(3t^2+1)^2}\,dt$ $\quad u=3t^2+1$, $du=6t\,dt$, $\frac{du}{6}=t\,dt$ $\quad\rightsquigarrow \int u^{-2}\frac{du}{6} = \frac{1}{6}\frac{u^{-1}}{-1}+C = \boxed{-\frac{1}{6}\left(\frac{1}{3t^2+1}\right)+C}$

4a) $f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$

b) for $f'(x)=\frac{3}{x}$, $f'(x)=\lim_{h\to0}\frac{\frac{3}{x+h}-\frac{3}{x}}{h} = \lim_{h\to0}\frac{1}{h}\frac{3x-3(x+h)}{(x+h)(x)} = \lim_{h\to0}\frac{1}{\cancel{h}}\frac{-3\cancel{h}}{(x+h)(x)}$

$\boxed{=\frac{-3}{x^2}}$ ✓

5. 

When $h=3$ feet
$h'(t)=.25$ ft/min
(a) Find $\frac{dV}{dt}$ at that instant

$V=\frac{1}{3}\pi r^2h$ $\quad \frac{r}{h}=\frac{2}{6}=\frac{1}{3}$; $r=\frac{1}{3}h$

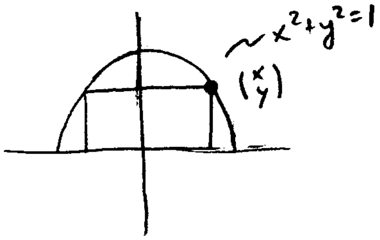
$\Rightarrow V=\frac{1}{27}\pi h^3$ $\quad\Rightarrow V'(t)=\frac{1}{27}\pi 3h^2h'$

$=\frac{\pi}{9}h^2h' = \frac{\pi}{9}(9)(.25) = \boxed{.25\pi \text{ ft}^3/\text{min}}$

5b) $V(6)-V(3)$
$=\frac{1}{27}\pi(6^3-3^3) = 7\pi \text{ ft}^3$

$T=\frac{\text{Vol}}{\text{rate}} = 7\pi/\frac{\pi}{4} = \boxed{28 \text{ min}}$

6.



maximize

$A = 2xy = 2x\sqrt{1-x^2}$ $\quad 0 \le x \le 1$ $\quad A(0)=A(1)=0$

So max is at a stationary pt (no sing pts in $(0,1)$)

$A'(x) = 2\sqrt{1-x^2} + 2x\,\frac{1}{2}(1-x^2)^{-1/2}(-2x)$

$= 2\sqrt{1-x^2} - \dfrac{2x^2}{\sqrt{1-x^2}}$

$A'(x)=0$ iff $\dfrac{x^2}{\sqrt{1-x^2}} = \sqrt{1-x^2} \Longrightarrow x^2 = 1-x^2$

$2x^2 = 1$

$\boxed{x = \frac{1}{\sqrt{2}}}$

so rectangle has width $2x = \sqrt{2}$
height $y = \frac{1}{\sqrt{2}}$
and area $2xy = 1$

7. $f(x) = \dfrac{x-3}{2x+2} = \dfrac{x-3}{2(x+1)}$

$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \dfrac{\cancel{x}[1-3/x]}{\cancel{x}[2+2/x]} = \dfrac{1}{2}$

so $y = 1/2$ is horiz. asymptote

vert. asymp at $x=-1$ (denom $=0$)

$\lim_{x\to -1^-} \dfrac{x-3}{2(x+1)} = \dfrac{-4}{2(0^-)} = +\infty$

$\lim_{x\to -1^+} f(x) = -\infty$

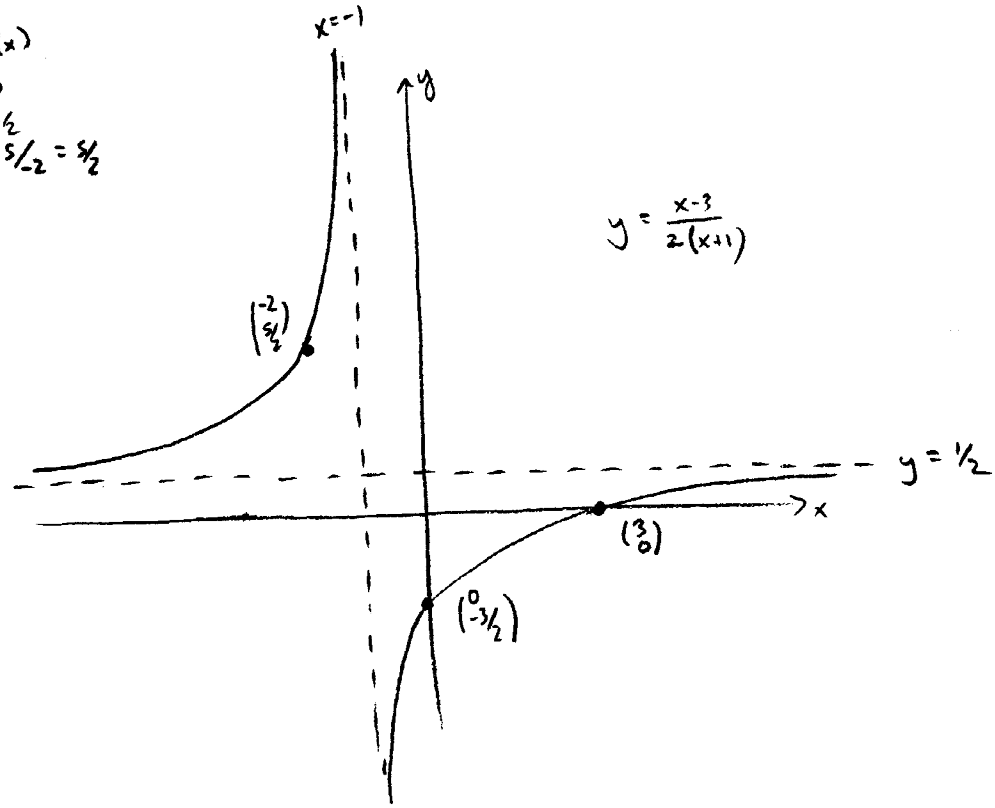
$f'(x) = \frac{1}{2}\,\dfrac{1(x+1)-(x-3)}{(x+1)^2} = \dfrac{2}{(x+1)^2}$

$f''(x) = \dfrac{-4}{(x+1)^3}$

+ + + DNE + + + sign f'
-1
f INC $\quad$ f INC

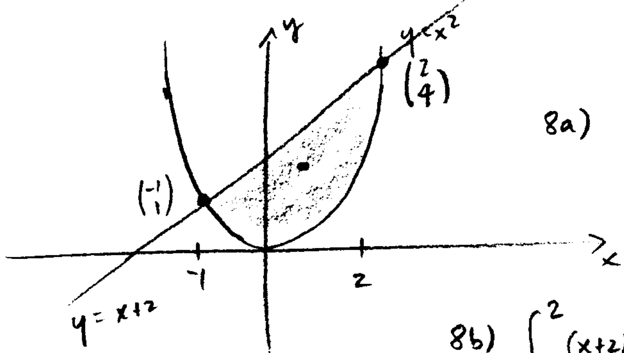
+ + + DNE - - - - sign f''
-1
f CU $\quad$ f CD

x	$f(x)$
+3	0
0	$-3/2$
-2	$-5/-2 = 5/2$



8). $y = x+2$
$y = x^2$

cross when
$x^2 = x+2$
$x^2 - x - 2 = 0$
$(x-2)(x+1) = 0$
$x = -1, 2$
$y = 1, 4$



8a)

8b) $\displaystyle\int_{-1}^{2} (x+2) - x^2\,dx = \left.\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$
$= \frac{1}{2}(4-1) + 2(2+1) - \frac{1}{3}(8+1)$
$= 3/2 + 6 - 3 = 4\frac{1}{2} = 9/2$

8c) $\delta = 1.$ $m = A = 4\frac{1}{2}$ from 8b)

$M_y = \displaystyle\int_{-1}^{2} x(x+2-x^2)\,dx = \int_{-1}^{2} x^2+2x-x^3\,dx = \left.\frac{x^3}{3} + x^2 - \frac{x^4}{4}\right]_{-1}^{2} = \frac{8}{3} + \cancel{4} - \cancel{4} - \left(-\frac{1}{3} + 1 - \frac{1}{4}\right)$
$= 3 - 1 + \frac{1}{4} = 2\frac{1}{4} = 9/4$

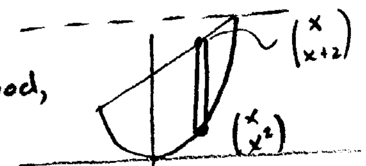
so $\bar{x} = \dfrac{M_y}{m} = \dfrac{9/4}{9/2} = \dfrac{2}{4} = \boxed{\frac{1}{2}}$

$M_x = \dfrac{1}{2}\displaystyle\int_{-1}^{2} (x+2)^2 - (x^2)^2\,dx = \frac{1}{2}\int_{-1}^{2} -x^4 + x^2 + 4x + 4\,dx = \frac{1}{2}\left[-\frac{x^5}{5} + \frac{x^3}{3} + 2x^2 + 4x\right]_{-1}^{2}$
$= \frac{1}{2}\left\{-\frac{1}{5}(32 - -1) + \frac{1}{3}(8 - -1) + 2(4-1) + 4(2 - -1)\right\}$
$= \frac{1}{2}\left\{-\frac{33}{5} + 3 + 6 + 12\right\} = \frac{1}{2}\left(\frac{-33+105}{5}\right) = \frac{1}{2}\,\frac{72}{5} = \boxed{7.2}$
whew!

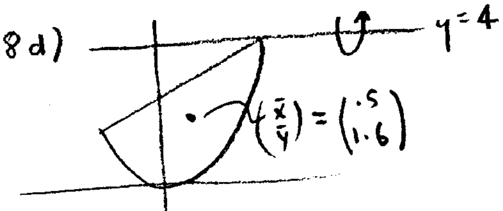
so $\bar{y} = \dfrac{M_x}{m} = \dfrac{7.2}{4.5} = \dfrac{72}{45} = \dfrac{8}{5} = \boxed{1.6}$

$M_y = 9/4 = 2.25$
$\bar{x} = .5$
$M_x = 7.2$
$\bar{y} = 1.6$

8e) washers good, at least for setup



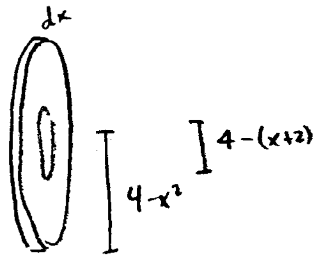
8d)



$V = (2\pi R)(A)$
$= 2\pi(4-1.6)(4.5)$
$= 2\pi(2.4)(4.5)$
$\boxed{V = 21.6\pi}$

$V = \displaystyle\int_{-1}^{2} \pi(x^4 - 9x^2 + 4x + 12)\,dx$
$= \pi\left.\left(\frac{x^5}{5} - 3x^3 + 2x^2 + 12x\right)\right]_{-1}^{2}$
$= \pi\left[\frac{1}{5}(32 - -1) - 3(8 - -1) + 2(4-1) + 12(2 - -1)\right]$
$= \pi\left[\frac{33}{5} - 27 + 6 + 36\right]$
$= \pi[6.6 + 15] = 21.6\pi$!

Yipes!



$dV = \pi\left((4-x^2)^2 - (4-(x+2))^2\right)dx$
$= \pi\left(\cancel{16} - 8x^2 + x^4 - \cancel{16} - (x^2+4x+4) + 8(x+2)\right)dx$

$dV = \pi(x^4 - 9x^2 + 4x + 12)\,dx$