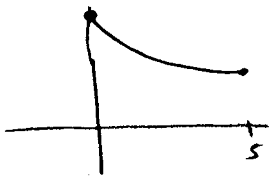
Math 1210-3/4
Practice exam solutions
3/7/03

p157 37) $t=0 \sim 5$ years ago
$f(t) = \#$ of dropouts
$f'(t)<0$ but $(f'(t))' > 0$



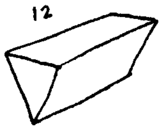
40) $V = \frac{4}{3}\pi r^3$

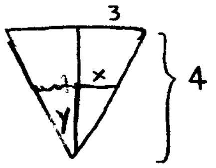
$\Rightarrow V'(t) = 4\pi r^2 r'(t)$

when $r=5$, $V'=10$

$10 = 100\pi r'(t) \Rightarrow r'(t) = \frac{1}{10\pi}$ m/h

41)





Volume of water $= xy \cdot 12$ ft^3

similar Δ's $\Rightarrow \frac{x}{y} = \frac{3}{4}$; $x = \frac{3}{4}y$

$V = 12\left(\frac{3}{4}\right)y^2 = 9y^2$

$\frac{dV}{dt} = 9$ ft³/min

Find $\frac{dy}{dt}$ when $y=3$

$V'(t) = 18yy'(t)$

$9 = 18 \cdot 3 \cdot y' \Rightarrow \boxed{y'(t) = \frac{1}{6} \text{ ft/min}}$

46) $y^2 = 4x^3$
$2yy' = 12x^2$
$y' = \frac{6x^2}{y}$
at $\binom{x}{y} = \binom{1}{2}$
$y' = \frac{6}{2} = 3$
$m_1 = 3$

$2x^2 + 3y^2 = 14$
$4x + 6yy' = 0$
$y' = -\frac{2x}{3y}$
at $\binom{x}{y} = \binom{1}{2}$
$y' = -\frac{2}{6}$
$m_2 = -1/3$

$m_1 m_2 = -1$ so tangent lines are ⊥

47) $y = \sin(\pi x) + x^2$

$dy = (\pi\cos\pi x + 2x)\,dx$

$x=2$
$dx=.01$

$dy = (\pi\cos 2\pi + 4)(.01)$

$\boxed{dy = (.01)(4+\pi)}$

$\approx .0714$

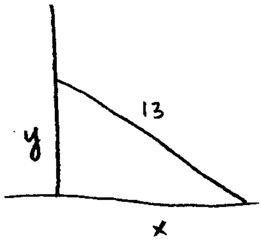
45d) $x\sin(xy) = x^2+1$

$\sin(xy) + x\cos(xy)[y+xy']$
$= 2x \;\; 1$

$y'[x^2\cos xy] = 2x \;\; 1 - \sin xy - xy\cos xy$

$\boxed{y' = \frac{2x - \sin xy - xy\cos xy}{x^2\cos xy}}$

50)



$x^2+y^2 = 169$

find $\frac{dy}{dt}$ when $y=5$ (& $x=12$)

given: $\frac{dx}{dt} = 2$

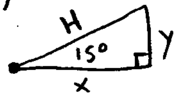
$2xx'(t) + 2yy'(t) = 0$

$xx' + yy' = 0$

$12 \cdot 2 + 5y' = 0$ $y' = -\frac{24}{5}$ ft/sec

so top is moving $\boxed{4.8 \text{ ft/sec}}$ down the wall, at that instant

51)



from $t=0$, $H(t) = 400t$ miles (since $H'(t) = 400$).

Find $y'(t)$.

$\frac{y}{H} = \sin 15° \Rightarrow y = H(\sin 15°)$

$\Rightarrow y'(t) = H'(t)\sin(15°)$

$= 400\sin(15°) \approx 104$ miles/hour

p203-204

8) $f(u) = u^2 (u-2)^{1/3}$; $[-1, 3]$

$$f'(u) = 2u(u-2)^{1/3} + u^2 \tfrac{1}{3}(u-2)^{-2/3} 1$$
$$= u(u-2)^{-2/3}\left[2(u-2) + \tfrac{1}{3}u\right]$$
$$\left[\tfrac{7}{3}u - 4\right]$$
$$\left[\tfrac{7}{3}\left(u - \tfrac{12}{7}\right)\right]$$

crit. pts:

endpoints -1 $f(-1) = 1(-1) = -1$

3 $f(3) = 9 \cdot 1 = 9$

sing pts 2 $f(2) = 0$

stat. pts $12/7$ $f\left(\tfrac{12}{7}\right) = \tfrac{144}{49}\left(-2/7\right)^{1/3} \approx$

20) $g(t) = t^3 + 1/t$

$g'(t) = 3t^2 - t^{-2} = t^{-2}\left[3t^4 - 1\right]$

$g''(t) = 6t + 2t^{-3}$ $\parallel$ $2t^{-3}\underbrace{\left[3t^4+1\right]}_{>0}$

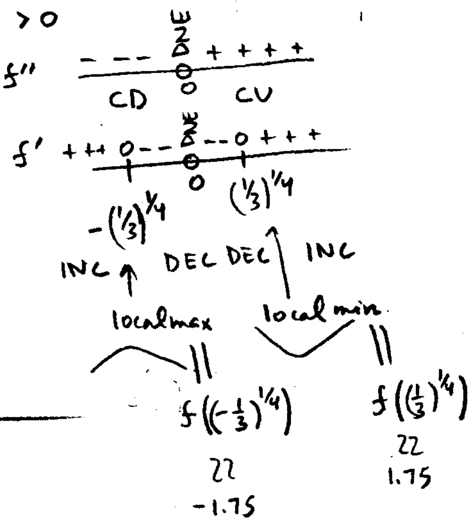
vert asymp $t = 0$

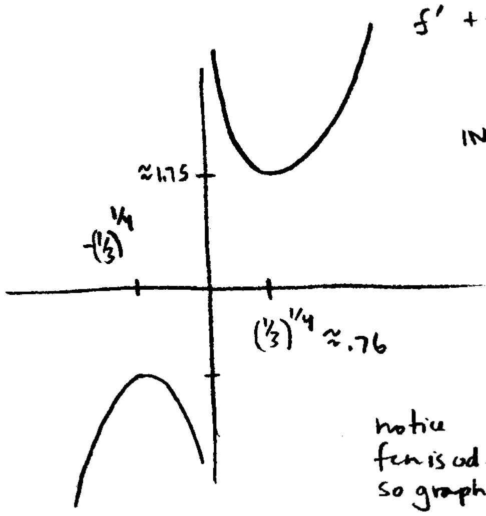
stat pts

$3t^4 = 1$

$t = \pm\left(\tfrac{1}{3}\right)^{1/4}$

<u>no</u> inflection pts!





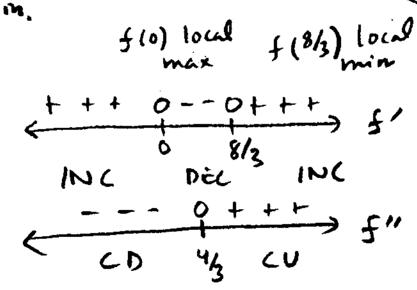
notice
fcn is odd,
so graph symm wrt origin.

21) $f(x) = x^2(x-4) = x^3 - 4x^2$

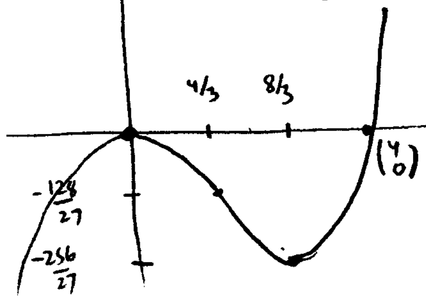
$f'(x) = 3x^2 - 8x = x(3x-8) = 3x(x - 8/3)$

$f''(x) = 6x - 8 = 6\left(x - \tfrac{4}{3}\right)$

x	$f(x)$	
0	0	← local max
8/3	$-\tfrac{256}{27}$	← local min
4/3	$-\tfrac{128}{27}$	infl. pt.
4	0	other x-intercept



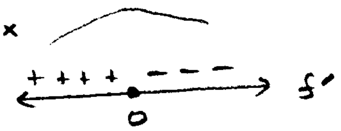
inflection pt $\begin{pmatrix} 4/3 \\ f(4/3) \end{pmatrix}$



22) $f(x) = \dfrac{4}{x^2+1} + 2$

$\lim_{x\to\pm\infty} f(x) = 0$ if f has max min values → must occur at stationary pts

$$f'(x) = 4(-1)(x^2+1)^{-2} 2x$$
$$= -\frac{8x}{(x^2+1)^2}$$



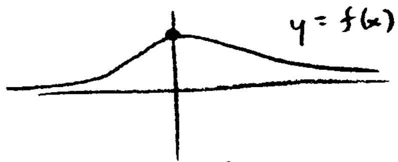
⇒ f has a local max at 0.

no other stat. pts & since $f \to 0$ as $x \to \pm\infty$

deduce

max value = $f(0) = 6$

<u>no</u> min value.



29) $f(x) = \dfrac{3x^2-1}{x} = 3x - \dfrac{1}{x}$

$x = 0$ vert. asymptote

$y = 3x$ diag asymptote

x-intercepts $x = \pm\frac{1}{\sqrt{3}}$

$f'(x) = 3 + \frac{1}{x^2} > 0$

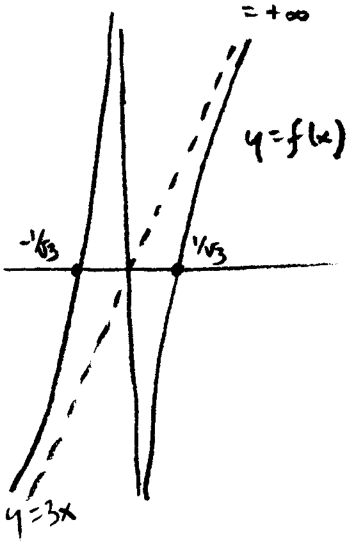
f INC on $(-\infty, 0)$, $(0, \infty)$

$f''(x) = -\frac{2}{x^3}$ f CU on $(-\infty, 0)$
CD on $(0, \infty)$

no local extrema.

$\lim_{x\to 0^+} f(x) = \frac{-1}{0^+}$
$= -\infty$

$\lim_{x\to 0^-} f(x) = \frac{-1}{0^-}$
$= +\infty$



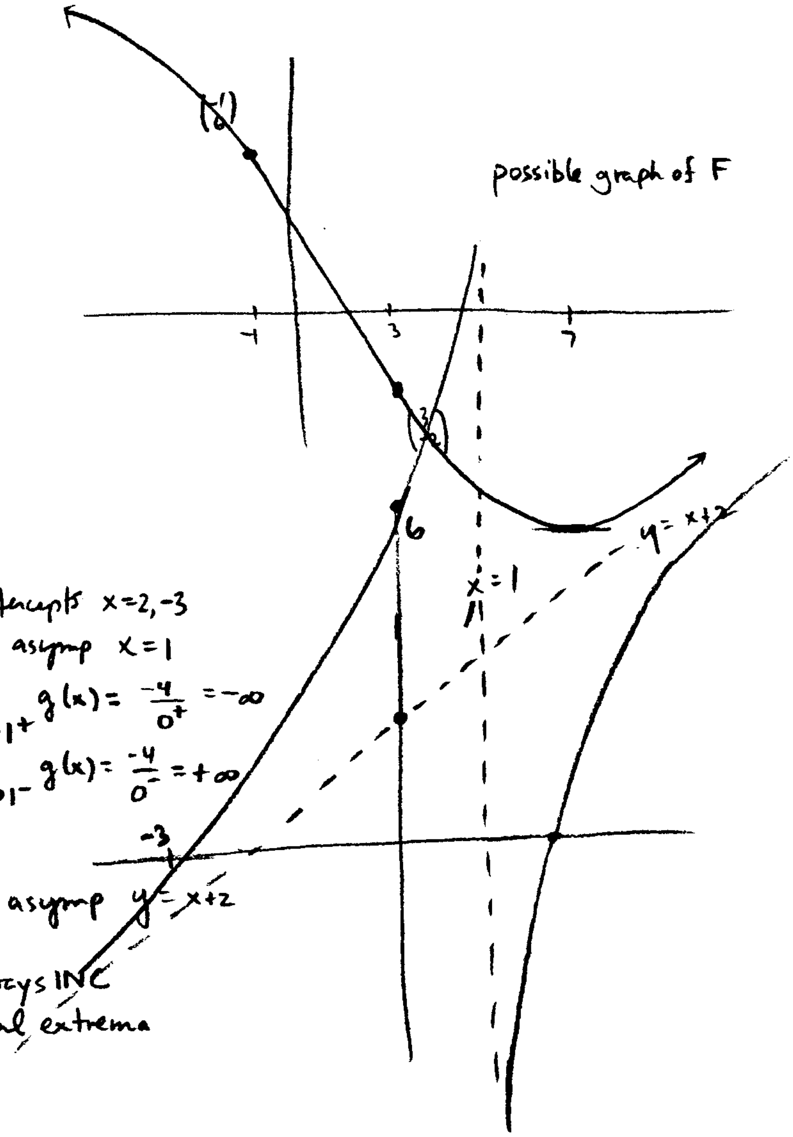
38)

x	$F(x)$	$F'(x)$
-1	6	-2
3	-2	-2
7		0

$F'<0$

$F''<0$ -1 $\quad F''=0$ 3 $\quad F''>0$

dec, CD $\quad$ F a line of slope -2 $\quad$ CU.



p. 196 # 17.

$g(x) = \frac{x^2+x-6}{x-1} = \frac{(x+3)(x-2)}{x-1}$

x intercepts $x=2,-3$

vert. asymp $x=1$

$\lim_{x\to 1^+} g(x) = \frac{-4}{0^+} = -\infty$

$\lim_{x\to 1^-} g(x) = \frac{-4}{0^-} = +\infty$

$$\begin{array}{r} x+2 \\ x-1 \overline{) x^2+x-6} \\ \underline{x^2-x} \\ 2x-3 \\ \underline{2x-2} \\ -1 \end{array}$$

so $g(x) = x+2 - \frac{1}{x-1}$ $\quad$ diag asymp $y = x+2$

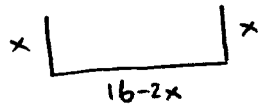
$g'(x) = 1 + \frac{1}{(x-1)^2} > 0$ ~ g always INC
no local extrema

$g''(x) = -2(x-1)^{-3}$

$>0 \quad x<1 \quad$ CU

$<0 \quad x>1 \quad$ CD

40.



maximize $A = x(16-2x)$
$= 16x - 2x^2$

$0 \le x \le 8$

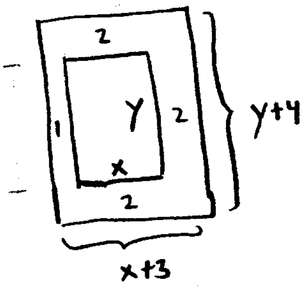
end pts $\quad A(0) = A(8) = 0$

stat pts $\quad A'(x) = 16 - 4x = 0$

$\boxed{\underline{\underline{x=4}}}$ $\quad A(4) = 4 \cdot 8 = 32$ in^2
max section area

no sing pts

42.



minimize $(x+3)(y+4) = A$;

print area $= xy = 27$

$y = \frac{27}{x}$

$A(x) = (x+3)\left(\frac{27}{x}+4\right)$ $\quad$ domain $x>0$

$= 27 + \frac{81}{x} + 4x + 12$

$A'(x) = -\frac{81}{x^2} + 4$

$= \frac{4x^2-81}{x^2} = \frac{(2x-9)(2x+9)}{x^2}$

$A'(x) = 0$ for $x = 9/2$

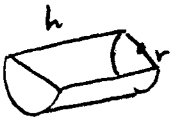
$- - 0 + + +$ $\quad$ A'

0 $\quad$ 9/2

$A(9/2)$ is min val.

so page dims are $\boxed{\begin{array}{l} \text{width} = \frac{9}{2}+3 = \frac{15}{2} \\ \text{ht} = \frac{27}{15/2} + 4 = \frac{38}{5} \end{array}}$ $= 7.5$, $= 7.6$

43. (this looks like half the can problem)



$V = 128\pi = \frac{1}{2}\pi r^2 h$; $h = \frac{256}{r^2}$

material $= \underbrace{2\left(\frac{1}{2}\pi r^2\right)}_{\text{ends}} + \underbrace{\frac{1}{2}(2\pi r)h}_{\text{sides}} = \pi r^2 + \pi r h$

minimize $f(r) = \pi r^2 + \pi r\left(\frac{256}{r^2}\right) = \pi\left[r^2 + \frac{256}{r}\right]$

$f'(r) = \pi\left(2r - \frac{256}{r^2}\right) = 0$ $\quad 2r = \frac{256}{r^2}$ $\quad r^3 = 128 = 4^3 \cdot 2$

$\boxed{r = 4\sqrt[3]{2}}$

$f''(r) = \pi\left(2 + \frac{512}{r^3}\right) > 0$

for $r > 0$

so stat pt is MIN

$h = \frac{256}{16 \cdot 2^{2/3}} = \frac{2^4}{2^{2/3}} = 2^{3\frac{1}{3}}$ $\boxed{8\sqrt[3]{2} = h}$

44. $f(x) = \frac{1}{4}(x^2 + 6x + 8)$ on $[-2,0]$

$-\frac{1}{6}(x^2 + 4x - 12)$ on $[0,2]$

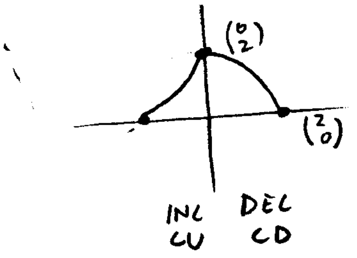
notice $f(0) = 2$ is well defined.

on $(-2,0)$ $f'(x) = \frac{1}{4}(2x+6) = \frac{1}{2}(x+3) > 0$ so f is increasing on $[-2,0]$

$f''(x) = \frac{1}{2} > 0$ so f is CU.

on $(0,2)$ $f'(x) = -\frac{1}{6}(2x+4) = -\frac{1}{3}(x+2) < 0$ on $(0,2)$ so f is decreasing on this int

$f''(x) = -\frac{1}{3}$ CD



x	f(x)
-2	0
0	2
2	0

min val = 0, at endpts

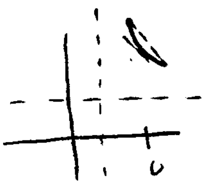
max val = 2 at sing pt $x = 0$

45c) $g(x) = \frac{x+1}{x-1}$; $I = [2,3]$

MVT applies.

$g'(x) = \frac{(x-1)-(x+1)}{(x-1)^2}$

$= \frac{-2}{(x-1)^2}$



$\frac{g(3)-g(2)}{1} = \frac{2-3}{1} = -1 = \frac{-2}{(x-1)^2}$

$(x-1)^2 = 2$

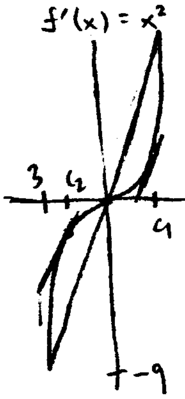
$x - 1 = \pm\sqrt{2}$, $x = \underline{1+\sqrt{2}}$ in interval.

45a) $f(x) = \frac{x^3}{3}$, $I = [-3,3]$ $\quad \frac{f(3)-f(-3)}{6} = \frac{18}{6} = 3$

$f'(x) = x^2$ $\quad f'(c) = 3$

$c^2 = 3$

$c = \pm\sqrt{3}$



45b) $f(x) = x^{3/5} + 1$; $[-1,1]$.

$f'(x) = \frac{3}{5}x^{-2/5}$

$\frac{f(1)-f(-1)}{2} = \frac{2}{2} = 1 = \frac{3}{5}x^{-2/5}$

$\Rightarrow x^{2/5} = 3/5$

$\Rightarrow x = \pm(3/5)^{5/2}$

MVT does not apply since $x=0$ is sing pt (even though conclusion holds)