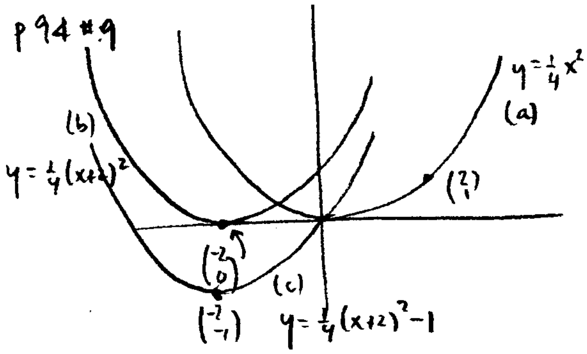
Math 1210-3/4

Practice Exam 1 solns

p 94 #9



#11 $F(x)=\sqrt{1+\sin^2 x}$

$k(x)=\sin x$

$h(y)=y^2$

$g(z)=1+z$

$f(w)=\sqrt{w}$

$F(x)=f(g(h(k(x))))$

$F'(x)=\frac{1}{2}(1+\sin^2 x)^{-\frac{1}{2}}(2\sin x\cos x)$

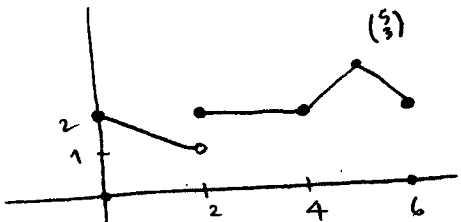
#19 $\lim_{x\to 2}\frac{1-2/x}{x^2-4}=\lim_{x\to 2}\frac{x-2/x}{(x-2)(x+4)}$

$=\lim_{x\to 2}\frac{\cancel{(x-2)}}{x\cancel{(x-2)}(x+4)}=\frac{1}{2\cdot 6}=\frac{1}{12}$

27 $\lim_{t\to 2^-}([\![t]\!]-t)=\lim_{t\to 2^-}[\![t]\!]-\lim_{t\to 2^-}t$

$=1-2=-1$

32.



p.97 Exercise 3

$f_1(x)=\sin(2x)+1$ period $=\pi$, vertical trans $=1$

Figure 1a

$f_2(x)=\sin(x+1)$ shift $y=\sin(x)$ 1 unit left

Figure 1d

$f_3(x)=3\sin(2x)$ amplitude $=3$ period $=\pi$

Figure 1c

$f_4(x)=3\sin(x+1)$ phase shift 1 unit left amplitude $=3$

figure 1e

$f_5(x)=3\sin x$ amplitude $=3$

figure 1b

p.80 #3 $\lim_{t\to 0}\frac{\cos^2 t}{1+\sin t}=\frac{1}{1+0}=1$

#8. $\lim_{\theta\to 0}\frac{\tan 5\theta}{\sin 2\theta}=\lim_{\theta\to 0}\frac{\sin 5\theta}{\sin 2\theta\cos 5\theta}$

$=\lim_{\theta\to 0}\underbrace{\frac{\sin 5\theta}{5\theta}}_{\downarrow\, 1}\ \underbrace{\frac{2\theta}{\sin 2\theta}}_{\downarrow\, 1}\ \underbrace{\frac{5}{2}\frac{1}{\cos 5\theta}}_{\downarrow\, 5/2}$

$=5/2$

#11 $\lim_{t\to 0}\frac{\tan^2 3t}{2t}$

$=\lim_{t\to 0}\frac{3}{2}\underbrace{\frac{\sin 3t}{3t}}_{\downarrow\, 1}\ \underbrace{\frac{\sin 3t}{\cos^2 3t}}_{\downarrow\, 0}=0$

p105-106

17. $m=\frac{1}{2}t^2+1$

(a) $m(2.01)-m(2)=\frac{1}{2}(2.01)^2-\frac{1}{2}2^2$

$=.02005$ g

(b) $\frac{m(2.01)-m(2)}{.01}=2.005$ g/hour

(c) $m'(t)=t$

$m'(2)=2$ g/hour

23.

$\frac{f(24)-f(0)}{24}\ \frac{g}{h}$

$=\frac{100-800}{24}$ thousands of g/h

$=-29{,}167$ g/h ; so water used at average rate of 29,167 g/h

tangent line thru pt at 8am has slope $\approx\frac{100-700}{8}\cdot 10^3\approx -75{,}000$

so rate at 8am $\approx 75{,}000$ g/h

p 156-157

1 a) $f(x)=3x^3$

$$f'(x)=\lim_{h\to 0}\frac{3(x+h)^3-3x^3}{h}=\lim_{h\to 0}\frac{3(\cancel{x^3}+3x^2h+3xh^2+h^3)-\cancel{3x^3}}{h}$$

$$=\lim_{h\to 0} 9x^2+9xh+3h^2=9x^2$$

c) $f(x)=\frac{1}{3x}$

$$\lim_{h\to 0}\frac{\frac{1}{3(x+h)}-\frac{1}{3x}}{h}=\lim_{h\to 0}\frac{1}{h}\left(\frac{1}{3}\right)\left(\frac{1}{x+h}-\frac{1}{x}\right)=\lim_{h\to 0}\frac{1}{3}\frac{1}{h}\frac{x-(x+h)}{(x+h)x}$$

$$=\lim_{h\to 0}\frac{1}{3}\underbrace{\frac{-h}{h}}_{=-1}\underbrace{\frac{1}{(x+h)(x)}}_{\to\frac{1}{x^2}}$$

$$=-\frac{1}{3x^2}$$

f) $f(x)=\sin 3x$

$$\lim_{h\to 0}\frac{\sin(3x+3h)-\sin 3x}{h}$$

$$=\lim_{h\to 0}\frac{\sin 3x\cos 3h+\cos 3x\sin 3h-\sin 3x}{h}$$

$$=\lim_{h\to 0}\sin 3x\underbrace{\left[\frac{\cos 3h-1}{h}\right]}_{\to 0\ (\text{see notes})}+\cos 3x\left[\frac{\sin 3h}{h}\right]=3\cos 3x$$

$$\frac{\sin 3h}{3h}\cdot 3,\quad \frac{\sin 3h}{3h}\to 1,\quad 3\to 3:\quad 1\cdot 3$$

notice in 1acf
I can check my answer
using differentiation rules:

(a) $D_x\, 3x^3=3\cdot 3x^2=9x^2$ ✓ power rule

b) $D_x\,\frac{1}{3}x^{-1}=\frac{1}{3}(-1)(x^{-2})=-\frac{1}{3x^2}$ ✓ power rule

c) $D_x \sin 3x=(\cos 3x)3$ ✓ chain rule.

3 c) $\frac{f(1+\Delta x)-f(1)}{\Delta x}$ where $f(z)=z^{3/2}$
so expression is $f'(1)$

h) $f(x)=\frac{1}{\sqrt{x}}$, at $x=5$ is one possibility

4. a) $f'(2)=$ slope of graph at $t=2\approx-\frac{3.5}{4.5}=-\frac{7}{9}\approx-.8$ $\left(\frac{\text{length units}}{\text{time units}}\right)$

b) $f'(6)=$ " " $t=6\approx 3$

c) $\frac{f(7)-f(3)}{7-3}=\frac{6-1.5}{4}=\frac{4.5}{4}=1.125$

d) $\frac{d}{dt}f(t^2)=f'(t^2)\cdot 2t$ at $t=2$
$=f'(4)\cdot 4\approx\frac{1}{2}\cdot 4=2$

e) $\frac{d}{dt}(f^2(t))=2f(t)f'(t)$ at $t=2$
$=2f(2)f'(2)$
$=2\cdot 2\cdot(-.8)\approx-3.2$

f) $\frac{d}{dt}f(f(t))$
$=f'(f(t))\cdot f'(t)$ at $t=2$
$=f'(f(2))\cdot f'(2)$
$=(f'(2))^2\approx .64$

5. $D_x(3x^5)=3\cdot 5x^4=15x^4$

11. $D_x\,\frac{4x^2-2}{x^3+x}=\frac{8x(x^3+x)-(4x^2-2)(3x^2+1)}{(x^3+x)^2}$

12. $D_t\left(t(2t+6)^{1/2}\right)=1\,(2t+6)^{1/2}+t\cdot\frac{1}{2}(2t+6)^{-1/2}\cdot 2$

22. $D_x\left(\frac{\sin 3x}{\cos(5x^2)}\right)=\frac{(\cos 3x)3\cos(5x^2)-(\sin 3x)(-\sin(5x^2))\cdot 10x}{\cos^2(5x^2)}$

39. $V = \frac{4}{3}\pi r^3$ $\frac{dV}{dr} = \frac{4}{3}\pi \cdot 3r^2 = 4\pi r^2$

40. $\frac{dV}{dt} = \frac{dV}{dr}\frac{dr}{dt}$

$10 = 4\pi 5^2 \cdot \frac{dr}{dt}$ so $\frac{dr}{dt} = \frac{10}{100\pi} = \frac{1}{10\pi}$ m/sec

42. $f(t) = 128t - 16t^2$ feet

(a) $f'(t) = 0$ at max ht $= 128 - 32t \Rightarrow t = 4$ sec

$f(4) = 128 \cdot 4 - 16 \cdot 16 = 256$ ft

(b) $f(t) = 0 = 16t(8-t)$

$t = 8$ sec object returns, vel. $= f'(8) = 128 - 32 \cdot 8$

$= -128$ ft/sec (note, opposite of initial velocity)

43. $f(t) = t^3 - 6t^2 + 9t$

(a) when $f'(t) < 0$. $f'(t) = 3t^2 - 12t + 9 = 3(t^2 - 4t + 3) = 3(t-3)(t-1)$

$t < 1$, $f'(t) = (\text{neg})(\text{neg}) = \text{pos}$

$1 < t < 3$: $f'(t) = (\text{neg})(\text{pos}) = \text{neg}$ $\Rightarrow$ object moving left $1 < t < 3$

$t > 3$ $f'(t) = (\text{pos})(\text{pos}) = \text{pos}$

(b) $f''(t) = 6t - 12$

$f''(1) = -6$

$f''(3) = 6$

(c) $f''(t) = 6(t-2) > 0$ when $t > 2$

at $x = 2$:

$f(2) = 3$ $g(2) = 2$

$f'(2) = 4$ $g'(2) = 5$

$f''(2) = -1$

49. a) $D_x\left(f^2(x) + g^3(x)\right) = 2ff' + 3g^2 g'$

at $x = 2$, get $2 \cdot 3 \cdot 4 + 3 \cdot 2^2 \cdot 5$

$= 24 + 60 = 84$

b) $(fg)' = f'g + fg'$

at $x = 2$ get $4 \cdot 2 + 3 \cdot 5 = 23$

c) $D_x(f(g(x))) = f'(g(x)) \cdot g'(x) =$

at $x = 2$ get $f'(2) \cdot 5 = 20$

d) $D_x(f^2(x)) = 2ff'$

$D_x(\text{"}) = 2(f'f' + ff'')$

at $x = 2$ get $2(4^2 + 3(-1)) = 26$