Math 1210-3/4
Wednesday April 23.

Review Sheet for Final

- review session Saturday 10 a.m. -12 noon JWB 335.
  We'll discuss practice exam & more.
  Final exam will address concepts, computations, applications

Key Concepts

Derivative
- average & instantaneous rate of change; geometric & physics interpretations
- precise limit definition
  $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Definite integral
- precise limit definition
  $$\int_a^b f(x)\,dx = \lim_{|P|\to 0} \sum_{i=1}^{n} f(\bar{x}_i)\Delta x_i$$

FTC, which relates these two fundamental concepts:
$$\int_a^b f(x)\,dx = F(b) - F(a) \quad \text{if } F'(x) = f(x) \ \forall x \in [a,b].$$

Quick Computations

limits (know limit theorems!)
differentiation (know all differentiation rules!)
antidifferentiation
definite integrals
substitution in integration

Applications (not all of these fit on one exam; I will need to make choices!)

velocity, acceleration, position
implicit differentiation
related rates
max-min
graphing
- INC, DEC, concavity, asymptotes, extrema

separable DE's
area between curves
average value
volumes
- slicing (slabs), including disks & washers
- cylindrical shells

curve length
surface area of revolution
work
moments, centers of mass, Pappus.

Math 1210-3
Practice Final Exam
April 23, 2003

Please show all work and reasoning for full credit. This exam is closed book and closed note, but you may use a scientific calculator. You may not use a calculator which does graphing or symbolic differentiation, however. I have provided you with the integral tables, geometry identities and some summation formulas. There are 200 points possible, as indicated below and in the exam. You have two hours for this exam so apportion your time accordingly. Good Luck!!

This is a practice exam - it is meant to indicate the possible range of problems and difficulty you might expect on the actual exam. Of course, there are topics which are not represented here but which may still appear on the actual exam, so you should look over your old exams, your homework, and your notes.

1) Compute the following limits

1a)

$$\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3}$$

(5 points)

1b)

$$\lim_{t \to 1^-} \frac{t^2 + 1}{t^2 - 1}$$

(5 points)

1c)

$$\lim_{x \to \infty} \frac{3x - x^2}{5x^2 + 2x + 53}$$

(5 points)

2) Compute the following derivatives

2a)

$$D_x\left(24x^2 + 12\frac{1}{x^2 + 1}\right)$$

(5 points)

2b)

$$D_t(t\sqrt{3t + 7})$$

(5 points)

2c)

$$\left[\frac{d}{dx}\right]\left[\frac{[\sin(2x)]^5}{\cos(3x^2)}\right]$$

(5 points)

2d) Suppose $f(1)=5$, $g(1)=1$, $f'(1)=3$, $g'(1)=-2$. Compute the derivative of the function

$8\,f(x)+g(x)^3+f(g(x))$, at $x=1$.

(10 points)

3) Compute the following integrals

3a)

$$\int 7\,u^3+3\sin(u)+2\frac{1}{u^2}\,du$$

(7 points)

3b)

$$\int_0^{1/2\,\pi} \cos(x)\sin(x)\,dx$$

(8 points)

3c)

$$\int_{-2}^{3} (x^2+1)^2\,dx$$

(8 points)

3d)

$$\int \frac{t}{(3\,t^2+1)^2}\,dt$$

(7 points)

4a) Write, for a funtion f(x), the limit definition of the derivative $f'(x)$.

(5 points)

4b) Use the limit definition of derivative to compute the derivative of

$$f(x)=3\frac{1}{x}$$

(15 points)

5) A mountain cabin has a drinking-water cistern, shaped like an inverted cone. the depth is 6 feet, and the radius of the circular top is 2 feet. The cistern is filled with water from a spring. After begin totally drained the cistern is being refilled. When the depth of the water is 3 feet it is increasing at a rate of 3 inches per minute.

5a) At what rate is water flowing into the cistern, at that instant?

(15 points)

5b) Assuming that the inflow rate remains constant and that no one is using water from the cistern, how much later will the cistern be completely filled?

(5 points)

6) A rectangle is to be drawn with horizontal and vertical sides, so that the two top corners touch the upper half of the unit circle centered at the origin, and so that the bottom two corners and bottom edge lie along the x-axis. Find the rectangle of maximum area which satisfies these constraints. (20 points)

7) Graph the function

$$f(x)=\frac{x-3}{2x+2}$$

Include the following information: horizontal and vertical asymptotes, intervals on which f is increasing and decreasing, concavity information, local extrema, intercepts. (20 points)

8) Consider the region bounded by the line $y=x+2$ and the graph of $y=x^2$.

8a) Sketch the region. Find the coordinates of the points where the curves cross. (5 points)

8b) Find the region's area. (5 points)

8c) Find the moments and center of mass of this region (assuming a constant density of 1). (20 points)

8d) The region in part 8a) is rotated about the line y=4. Use Pappus' Theorem and your work in parts 8b) and 8c) to deduce the volume of the resulting solid. (5 points)

8e) Verify that Pappus gives the correct result by reworking the volume for the solid of revolution in part 8d, using either disks, washers, or cylindrical shells. (Only one method works really well here, and the integral is somewhat lengthy but straightforward to compute. Probably on the actual exam I would find a shorter integral, or only ask you to set it up and not work it out.) (15 points)