$n = 70?$

Name ___Solutions___

Student I.D. ____________________

Math 1210-3
Exam #2 retake
March 28, 2003

Please show all work for full credit. This exam is closed book and closed note, but you may use a scientific calculator. You may not use a calculator which does graphing or symbolic differentiation, however. On the last page of the exam there is a table with geometry and trigonometry formulas. There are 100 points possible, as indicated below and in the exam. Since you only have 50 minutes you should be careful not to spend too long on any one problem. Good Luck!!

Score

	POSSIBLE
1______	25
2______	25
3______	20
4______	30
TOTAL______	100

1) Compute the following. Your reasoning should be clear for full credit.

1a)

$$\lim_{x \to 1^+} \frac{3x-5}{x^2-x}$$

(5 points)

$= \lim_{x \to 1^+} \frac{3x-5}{x(x-1)}$

$= \frac{-2}{1 \cdot 0^+} = \boxed{-\infty}$

1b) Find all horizontal or diagonal (oblique) asymptotes, if any, to the graph of

$$y = \frac{4x^2-1}{x-1}$$

(5 points)

$$\begin{array}{r} 4x+4 \\ x-1 \overline{) 4x^2-1} \\ 4x^2-4x \\ \hline 4x-1 \\ 4x-4 \\ \hline 3 \end{array}$$

so $y = 4x+4+\frac{3}{x-1}$

as $|x| \to \infty$, $\frac{3}{x-1} \to 0$

So we have a diagonal asymptote

$\boxed{y = 4x+4}$

1c) Use implicit differentiation to first find the slope, and then find the equation of the tangent line to the curve

$$x y^2 = 5x - 1$$

at the point $(x, y) = (1, -2)$.

(15 points)

$y = f(x)$

LHS = RHS

so $D_x(\text{LHS}) = D_x(\text{RHS})$

$1 \cdot y^2 + x\, 2y y' = 5$

$2xyy' = 5 - y^2$

$y' = \frac{5-y^2}{2xy}$

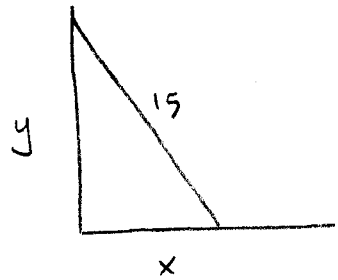
@ $(1, -2)$ $y' = \frac{5-4}{2(-2)} = -\frac{1}{4}$ = slope

So eqtn of the tangent line is $\boxed{(y+2) = -\frac{1}{4}(x-1)}$

(or $y = -\frac{1}{4}x - \frac{7}{4}$)

2) A 15-foot ladder is leaning against a vertical wall. If the bottom of the ladder is being pulled away from the wall at a constant rate of 3 feet per second, how fast is the top end of the ladder moving down the wall when it is 12 feet above the ground? Make sure to include correct units in your answer, and check your sign.

(25 points)



$$\boxed{\begin{array}{l} x'(t) = 3 \text{ ft/sec} \\ \text{find } y'(t) \text{ when } y = 12 \end{array}}$$

(ans will be $-y'(t)$).

$$x^2 + y^2 = 225$$

$$\Rightarrow \quad 2xx'(t) + 2yy'(t) = 0$$

$$xx' + yy' = 0$$

when $y = 12$
$x = 9 = \sqrt{225 - 144}$

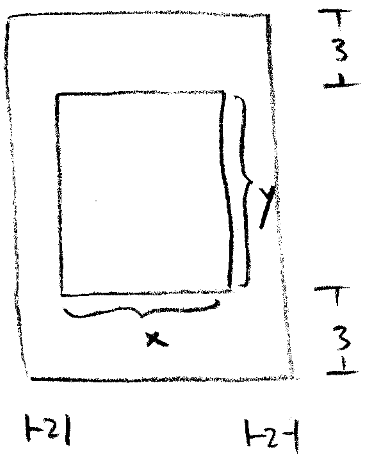
so $9 \cdot 3 + 12y' = 0$

$$y' = \frac{-27}{12} = -\frac{9}{4} = -2.25 \text{ ft/sec.}$$

so top of ladder is moving down wall at rate of $\boxed{2.25 \text{ ft/sec}}$

3) A rectangular handbill will require 54 square inches of area for the printed matter. You have decided to have vertical margins of 2 inches on each side, and 3 inch margins at the top and bottom. What size sheet of paper with these constraints has the smallest area?

(20 points)



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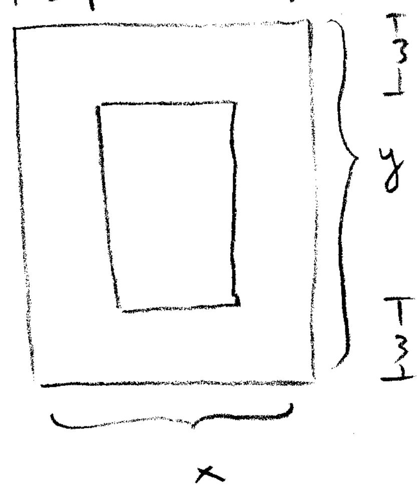


$xy = 54$

minimize $(x+4)(y+6)$

$= xy + 6x + 4y + 24$

$y = \frac{54}{x}$, so minimize

$f(x) = 54 + 6x + \frac{216}{x} + 24$, $0 < x < \infty$

$f'(x) = 6 - \frac{216}{x^2} = 0$, $f''(x) = \frac{432}{x^3} > 0$ on domain so stat pt gives min value

$6x^2 = 216$

$x^2 = 36$

$\boxed{\begin{matrix} x = 6 \\ y = 9 \end{matrix}}$

so paper dims are

$\boxed{\begin{matrix} \text{width} = 10'' \\ \text{height} = 15'' \end{matrix}}$

$(x-4)(y-6) = 54$

minimize xy

$y - 6 = \frac{54}{x-4}$

$y = \frac{54}{x-4} + 6$

so minimize

$f(x) = x\left[\left(\frac{54}{x-4}\right) + 6\right]$

$= \frac{54x}{x-4} + 6x$

so

$f'(x) = \frac{54[(x-4) - x]}{(x-4)^2} + 6$

$0 = \frac{-216}{(x-4)^2} + 6$

$\frac{216}{(x-4)^2} = 6$

$36 = (x-4)^2$

$6 = x - 4$

$\boxed{\begin{matrix} x = 10'' \\ y = 15'' \end{matrix}}$

4) Consider the function

$$f(x)=3\,x^4-4\,x^3$$

4a) On what intervals is f increasing and decreasing?

(5 points)

$f'(x) = 12x^3 - 12x^2$

$= 12x^2(x-1)$

$- - - - - 0 - - - 0 + + + +$ sign $f'(x)$

$0 \quad 1$

f DEC   f INC.

4b) On what intervals is f concave up and concave down?

(5 points)

$f''(x) = 36x^2 - 24x$

$= 12x(3x-2)$

$+ + + + 0 - 0 + + + +$ sign $f''(x)$

$0 \quad 2/3$

CU   CD   CU

4c) Identify where all local extrema of f occur, find their values, and identify whether they are local maxima or local minima.

(5 points)

$f(1) = 3-4 = -1$ is a local <u>min</u>

$f'<0 \quad 1 \quad f'>0$

4d) Find the x intercepts for the graph of f.

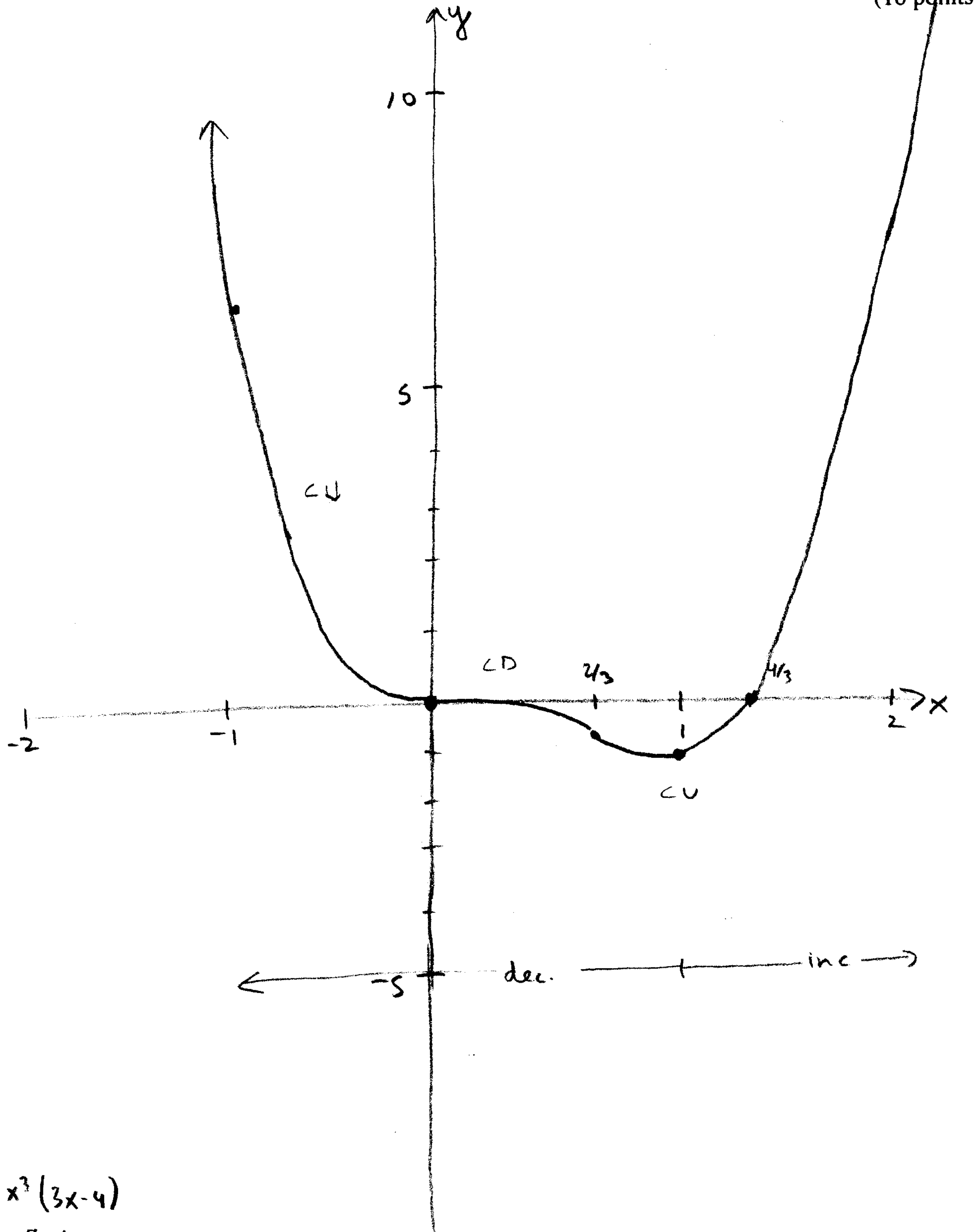
(5 points)

$f(x) = 0 = x^3(3x-4)$

$x = 0$

$x = \frac{4}{3}$

4d) Create an accurate graph for y=f(x), using all of your results from 4a-4d. (10 points)



x	$f(x)$
0	0
4/3	0
1	-1
2/3	$-\frac{16}{27}$
-1	7
2	16

$f(x) = x^3(3x-4)$

$f\left(\frac{2}{3}\right) = \frac{8}{27}(-2)$

$= -\frac{16}{27}$