

Name SOLUTIONS

Student I.D. _____

Math 1210-3
Exam #2
March 10, 2003

Please show all work for full credit. This exam is closed book and closed note, but you may use a scientific calculator. You may not use a calculator which does graphing or symbolic differentiation, however. On the last page of the exam there is a table with geometry and trigonometry formulas. There are 100 points possible, as indicated below and in the exam. Since you only have 50 minutes you should be careful not to spend too long on any one problem. Good Luck!!

Score	POSSIBLE
1 _____	20
2 _____	30
3 _____	20
4 _____	30
TOTAL _____	100

1) Compute the following. Your reasoning should be clear for full credit.

1a)

$$\lim_{x \rightarrow \infty} \frac{-3x^2 + 2}{x^2 - 4}$$

$$= \lim_{x \rightarrow \infty} \frac{x^2 \left[\frac{-3 + 2/x^2}{1 - 4/x^2} \right]}{x^2 \left[\frac{1 - 4/x^2}{1 - 0} \right]} = \frac{-3 + 0}{1 - 0} = \boxed{-3} \quad (5 \text{ points})$$

1b)

$$\lim_{x \rightarrow 2^+} \frac{-3x^2 + 2}{x^2 - 4}$$

$$= \lim_{x \rightarrow 2^+} \frac{-3x^2 + 2}{(x-2)(x+2)} = \frac{-10}{0^+ \cdot 4} = \boxed{-\infty} \quad (5 \text{ points})$$

1c) Find the slope of the tangent line to the curve

$$\cos(xy) = \frac{1}{2}$$

at the point on the curve with $x = \frac{1}{3}$ and $y = \pi$.

$$-\sin(xy) [1 \cdot y + x y'] = 0 \quad (10 \text{ points})$$

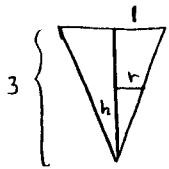
$$y' [-x \sin(xy)] = y \sin(xy)$$

$$\boxed{y' = -\frac{y}{x}} \quad @ \quad \left(\frac{1}{3}, \pi\right) \quad \boxed{y' = -\frac{\pi}{1/3} = -3\pi}$$

2) A water cistern in the shape of an inverted cone is being filled with water from a spring. The top of the tank is shaped in a circle, with diameter equal to 2 meters. The tank is 3 meters tall. When the water in the cone is 2 meters deep the depth is increasing at a rate of 9 centimeters per minute.

2a) At what rate is water flowing into the cistern? Make sure to include units in your answer.

(25 points)



$$\frac{r}{h} = \frac{1}{3} \quad (\text{similar } \Delta\text{'s})$$

When $h=2$, $\frac{dh}{dt} = .09 \text{ m/min}$

Find $\frac{dV}{dt}$.

$$V = \frac{1}{3} \pi r^2 h \quad \text{m}^3$$

$$r = \frac{1}{3} h$$

$$\Rightarrow V = \frac{1}{27} \pi h^3$$

$$V(t) = \frac{1}{27} \pi h(t)^3$$

$$V'(t) = \frac{1}{9} \pi h(t)^2 h'(t)$$

$$\textcircled{a} \quad h=2 \quad h'=.09 \Rightarrow V'(t) = \frac{1}{9} \pi \cdot 4 \cdot (.09) = .04\pi \text{ m}^3/\text{min}$$

$$= \frac{4\pi \times 10^4}{40\pi} \frac{\text{cm}^3}{\text{min}}$$

(5 points)

2b) If the water keeps flowing into the tank at this rate, how long will it take from the time the water is 2 meters deep until the tank is full?

$$V(h) = \frac{1}{27} \pi h^3$$

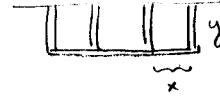
$$V(3) - V(2) = \frac{1}{27} \pi [27 - 8] = \frac{19}{27} \pi \text{ m}^3$$

$$\text{Time} = \frac{\text{Vol}}{\text{rate}} = \frac{\frac{19}{27} \pi}{.04\pi} = \frac{19}{1.08} \text{ minutes}$$

$$\approx 17.6 \text{ minutes}$$

3) Farmer Sally is fencing off three congruent and adjacent rectangular plots next to the road, to compare and advertise three new genetically engineered strains of corn. She will not put fencing along the road. She has bought 1200 feet of fencing. Use calculus to find the dimensions she should make her rectangular plots, in order to maximize their area, given that she uses the fencing she bought.

(20 points)



$$3x + 4y = 1200$$

$$\text{maximize } 3xy = A$$

$$3x = 1200 - 4y$$

$$A = y(1200 - 4y) = -4y^2 + 1200y$$

$$0 \leq y \leq \frac{1200}{4} = 300$$

$$A(0) - A(300) = 0$$

max at stat. pt.

$$A'(y) = -8y + 1200 = 0$$

$$8y = 1200$$

$$y = \frac{1200}{8} = 150$$

individual plots
 $y = 150$
 $x = 200$

$$x = 400 - \frac{4}{3}y = 400 - 200$$

$$\text{or } y = \frac{1200 - 3x}{4}$$

$$= 300 - \frac{3}{4}x$$

$$A = x \left[300 - \frac{3}{4}x \right]$$

$$0 \leq x \leq 400$$

$$A = 300x - \frac{3}{4}x^2$$

$$A'(x) = 300 - \frac{3}{2}x = 0$$

$$\frac{3}{2}x = 300$$

$$\frac{x}{2} = 100$$

$$x = 200$$

$$y = 300 - \frac{3}{4}(200)$$

$$= 150$$

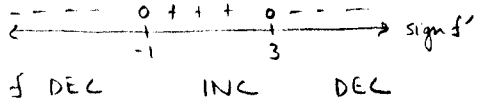
Same answer as

4) Consider the function

$$f(x) = -x^3 + 3x^2 + 9x$$

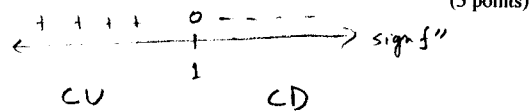
4a) On what intervals is f increasing and decreasing? (5 points)

$$\begin{aligned} f'(x) &= -3x^2 + 6x + 9 \\ &= -3(x^2 - 2x - 3) \\ &= -3(x-3)(x+1) \end{aligned}$$



4b) On what intervals is f concave up and concave down? (5 points)

$$\begin{aligned} f''(x) &= -6x + 6 \\ &= -6(x-1) \end{aligned}$$



$$\text{inf pt } \left(\frac{1}{-1+3+9} \right) = \left(\frac{1}{11} \right)$$

4c) Identify where all local extrema of f occur, find their values, and identify whether they are local maxima or local minima. (5 points)

$x = -1$ is a local min location
 value = $f(-1) = 1 - 3 - 9 = -11$
 $f'(-1) = 0$
 1st deriv test 4a, or 2nd deriv test 4b

local max at $x = 3$
 1st or 2nd deriv test
 value $f(3) = -27 + 27 + 27 = 27$

4d) Find the x intercepts for the graph of f . (5 points)

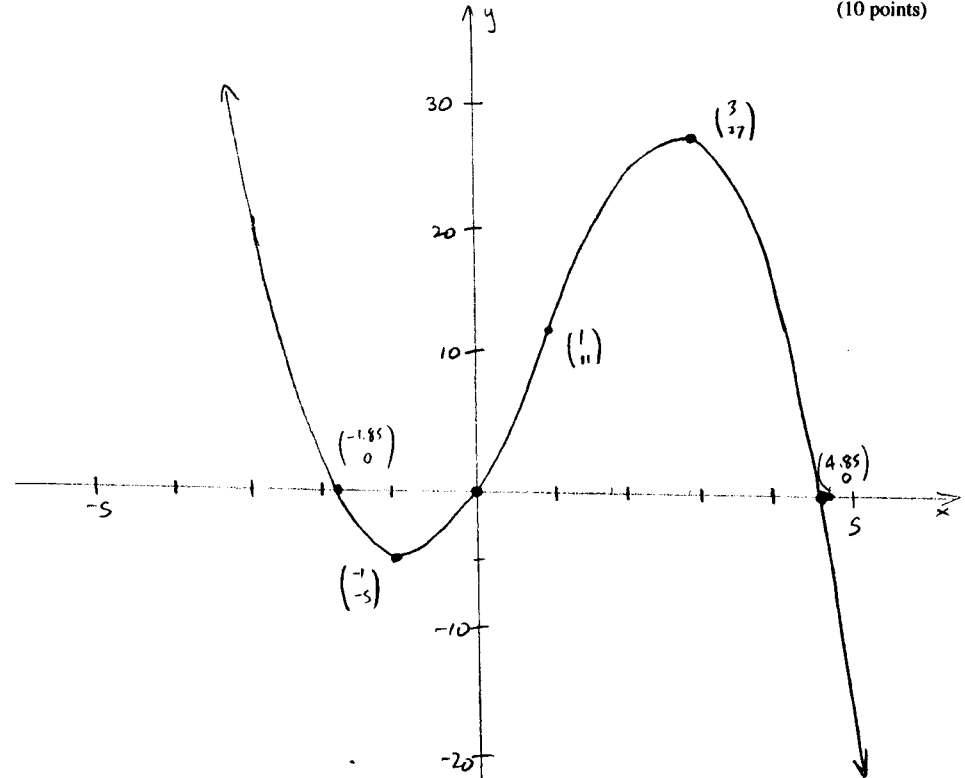
$$\begin{aligned} f(x) = 0 &= x(-x^2 + 3x + 9) \\ x = 0, \quad x &= \frac{-3 \pm \sqrt{9+36}}{-2} \\ x &= \frac{3(1 \pm \sqrt{5})}{2} \\ &\approx -1.85, 4.85 \end{aligned}$$

$$\{0, -1.85, 4.85\}$$

↑ ↑
approx

Page 5

4d) Create an accurate graph for $y=f(x)$, using all of your results from 4a-4d. (10 points)



x	$f(x)$
0	0
$-\frac{3+\sqrt{5}}{2}$	0
$\frac{3-\sqrt{5}}{2}$	0
-1	-11
3	27
1	11

Page 6