Name SOLUTIONS

Student I.D. ____________

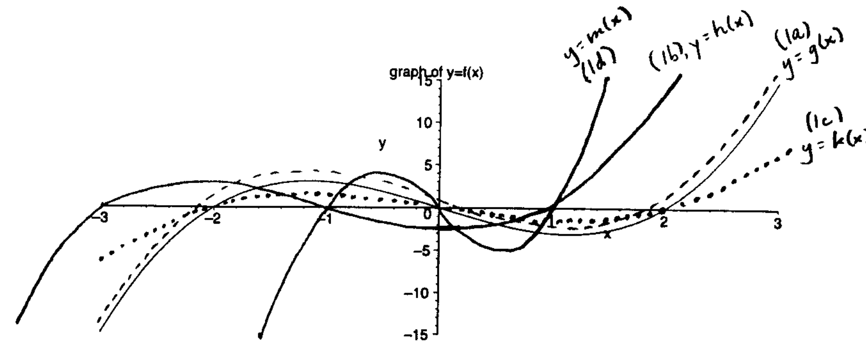
Math 1210-4
Exam #1
February 5, 2003

Please show all work for full credit. This exam is closed book and closed note, but you may use a scientific calculator. You may not use a calculator which does symbolic differentiation, however. There are 100 points possible, as indicated below and in the exam. Since you only have 50 minutes you should be careful to not spend too long on any one problem. Good Luck!!

Score

		POSSIBLE
1		20
2		20
3		25
4		25
5		10
TOTAL		

1) Here is the graph of a mystery function f(x):



Consider the four related functions below, and plot their grpahs onto the picture above. Indicate clearly which graph corresponds to which function, and explain how you obtained it fromthe graph of f.
(5 points each, for 20 total points)

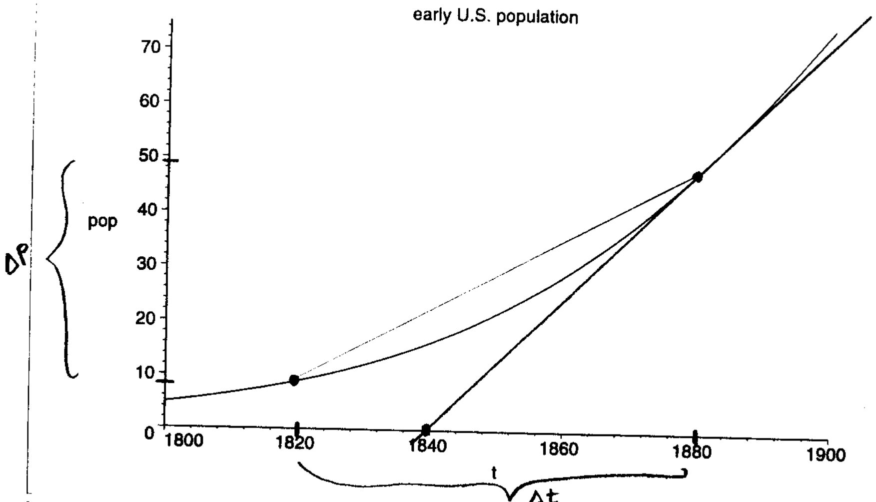
1a) $g(x)=f(x)+1$
shift up 1 unit

1b) $h(x)=f(x+1)$
move left 1 unit

1c) $k(x)=\frac{1}{2}f(x)$ scale vertically by factor of ½

1d) $m(x)=f(2x)$ compress horizontally by a factor of 2

2) Here is a plot of the population of the United States from 1800 to 1900. The vertical axis is marked off in millions of people.



2a) Use the graph to estimate the average population growth rate between 1820 and 1880. Include correct units!

(10 points)

$$\frac{\Delta P}{\Delta t} = \frac{P(1880)-P(1820)}{1880-1820} \approx \frac{48-8}{60} \times 10^6 \frac{\text{people}}{\text{years}}$$

$$= \tfrac{2}{3} \times 10^6 \text{ people/year}$$

$$\approx 670{,}000 \text{ people/year}$$

2b) Use the graph to estimate the (instantaneous) population growth rate in 1880.

(10 points)

= slope of tangent line to graph when $t = 1880$

$$\approx \frac{(48-0)\times 10^6}{1880-1840} \frac{\text{people}}{\text{year}}$$

$$= 1.2 \times 10^6 \text{ people/year}$$

$$= 1{,}200{,}000 \text{ people/year}$$

3) Compute the following derivatives

(25 points total)

3a) $D_t(t^3 + 8t^2 - 7)$

(8 points)

$$= 3t^2 + 16t$$

3b) $D_x y$ for $y = [\sin(3x)](x^3+1)$

(8 points)

$$D_x y = f'g + fg'$$

$$= (\cos 3x)3(x^3+1) + (\sin 3x)3x^2$$

3c) $D_x y$ for $y = \dfrac{(2x+1)^2}{4x^3 - 7x}$

(9 points)

$$D_x y = \frac{f'g - fg'}{g^2}$$

$$= \frac{2(2x+1)2(4x^3-7x) - (2x+1)^2(12x^2-7)}{(4x^3-7x)^2}$$

4a) Use the limit definition of derivative,

$$D_x f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

to compute the derivative of $f(x) = \frac{1}{2x+3}$.

(15 points)

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{2(x+h)+3} - \frac{1}{2x+3}}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{(2x+3) - [2(x+h)+3]}{[2(x+h)+3][2x+3]}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{\cancel{2x}+\cancel{3}-\cancel{2x}-2h-\cancel{3}}{[2(x+h)+3][2x+3]}\right]$$

$$= \lim_{h \to 0} \frac{-2\cancel{h}}{\cancel{h}} \frac{1}{[2(x+h)+3][2x+3]} = \boxed{-\frac{2}{(2x+3)^2}}$$

4b) Check your answer in part (4a) by using your favorite differentiation rules to compute the derivative of $f(x) = \frac{1}{2x+3} = (2x+3)^{-1}$

(5 points)

$$f'(x) = -(2x+3)^{-2}(2) = \frac{-2}{(2x+3)^2} \checkmark$$

4c) Find the equation of the tangent line to the graph of $y = \frac{1}{2x+3}$ passing through the point with x-coordinate equal to 1.

(5 points)

$$m = f'(1) = -\frac{2}{25}$$

$$f(1) = \frac{1}{5}, \quad (y - \underbrace{y_0}_{1/5}) = m(x - \underbrace{x_0}_{1}), \text{ so}$$

$$\boxed{\left(y - \frac{1}{5}\right) = -\frac{2}{25}(x-1)}$$

5) True/False. No justification needed.

(10 points total)

(a) The derivative of a polynomial is always a polynomial.

T, since $D_x x^n = nx^{n-1}$, & sum & constant rule [note: constants are degree zero polynomials

(b) The addition angle formula for cosine is: $\cos(x+y) = \cos(x)\sin(y) + \sin(x)\cos(y)$

F $\quad \cos(x+y) = \cos x \cos y - \sin x \sin y$

(c) For the number π, which is the area of the unit disk, the derivative of the function $f(x) = \pi^4$ is $D_x f(x) = 4\pi^3$

F $\quad f(x)$ is a constant so $D_x f(x) = 0$

(d) If a function is continuous at a point, then it must also be differentiable there.

F $\Leftarrow$ the reverse is true!

$y = |x|$ at $x = 0$

(e) If f(x) has derivative f'(2)=3, and value f(2)=5, then the derivative of the function $f(x)^4$ at x=2 is 1600.

$$D_x f(x)^4 = 4 f(x)^3 f'(x)$$

@ $x = 2$, $= 4(5^3)(3)$

$= 1500$

F