Name SOLUTIONS

Student I.D. ____________________

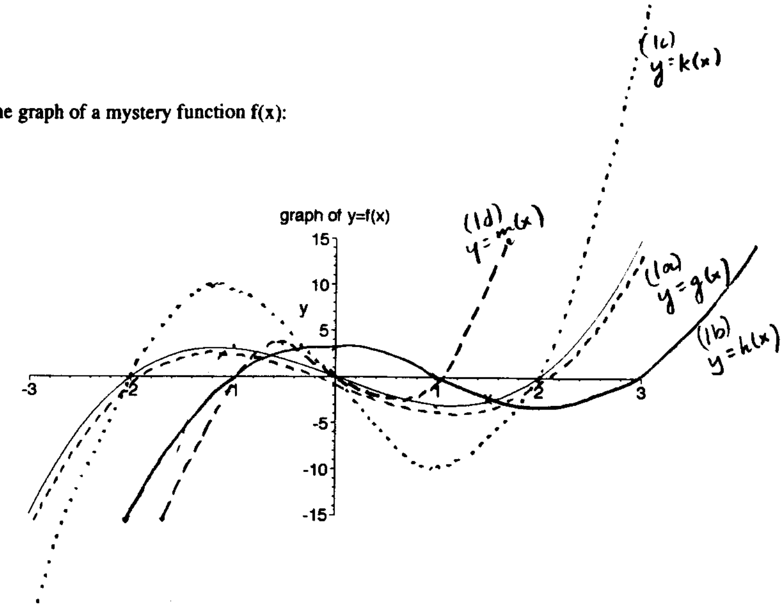
Math 1210-3
Exam #1
February 5, 2003

Please show all work for full credit. This exam is closed book and closed note, but you may use a scientific calculator. You may not use a calculator which does symbolic differentiation, however. There are 100 points possible, as indicated below and in the exam. Since you only have 50 minutes you should be careful to not spend too long on any one problem. Good Luck!!

Score

		POSSIBLE
1		20
2		20
3		25
4		25
5		10
TOTAL		

1) Here is the graph of a mystery function f(x):



Consider the four related functions below, and plot their graphs onto the picture above. Indicate clearly which graph corresponds to which function, and explain how you obtained it from the graph of f.

(5 points each, for 20 total points)

1a) $g(x)=f(x)-1$

shift down 1 unit

1b) $h(x)=f(x-1)$

shift right 1 unit

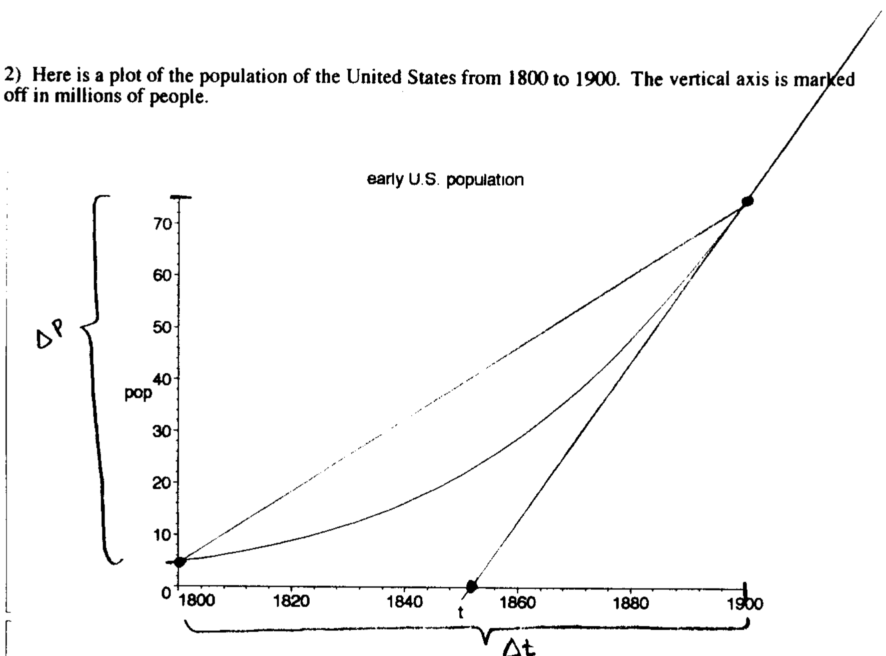
1c) $k(x)=3\,f(x)$

stretch vertically by a factor of 3

1d) $m(x)=f(2x)$

compress horizontally by a factor of 2

2) Here is a plot of the population of the United States from 1800 to 1900. The vertical axis is marked off in millions of people.



2a) Use the graph to estimate the average population growth rate between 1800 and 1900 (Include correct units and explanation ,in both 2a and 2b!)

10
(points)

$$\frac{\Delta P}{\Delta t} = \frac{P(1900)-P(1800)}{1900-1800} \ \frac{\text{people}}{\text{year}}$$

$$\approx \frac{(75-5)\times 10^6}{100} \ \text{people/year}$$

$$= 700{,}000 \ \text{people/year}$$

2b) Use the graph to estimate the (instantaneous) population growth rate in 1900.

10
(points)

= slope of tangent line to graph when $t = 1900$

$$\approx \frac{(75-0)\times 10^6}{1900-1852}$$

$$= \frac{75}{48} \times 10^6 \ \text{people/year}$$

$$= \frac{25}{16} \times 10^6$$

$$\approx 1{,}600{,}000 \ \text{people/year}$$

3) Compute the following derivatives

25
(points total)

3a) $D_t(t^3+8t^2-7)$

(8 points)

$$= 3t^2 + 16t$$

3b) $D_x y$ for $y=(x^2+1)\cos(3x)$

(8 points)

$$D_x y = f'g + fg'$$

$$= 2x\cos(3x) + (x^2+1)(-\sin 3x)3$$

3c) $D_x y$ for $y=\dfrac{4x^3-7x}{(2x+1)^2}$

(9 points)

$$D_x y = \frac{f'g - fg'}{g^2}$$

$$= \frac{(12x^2-7)(2x+1)^2 - (4x^3-7x)(2)(2x+1)^1 2}{(2x+1)^4}$$

4a) Use the limit definition of derivative,

$$D_x f(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

to compute the derivative of $f(x)=\frac{1}{x^2}$.

(15 points)

$$D_x f(x) = \lim_{h\to 0}\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$$

$$= \lim_{h\to 0}\frac{1}{h}\left(\frac{x^2-(x+h)^2}{(x+h)^2x^2}\right)$$

$$= \lim_{h\to 0}\frac{\cancel{x^2}-(\cancel{x^2}+2hx+h^2)}{h(x+h)^2x^2}$$

$$= \lim_{h\to 0}\frac{-2hx-h^2}{h(x+h)^2x^2}$$

$$= \lim_{h\to 0}\frac{-2x-h}{(x+h)^2x^2}$$

$$= \frac{-2x}{x^4} = \left|\frac{-2}{x^3}\right|$$

4b) Check your answer in part (4a), by using your favorite differentiation rule to compute the derivative of $f(x)=\frac{1}{x^2}$.

(5 points)

$f(x) = x^{-2}$

$f'(x) = -2x^{-3}$ ✓

4c) Find the equation of the tangent line to the graph of $y=\frac{1}{x^2}$, passing through the point with x-coordinate equal to 1.

(5 points)

$m = f'(1) = -2$

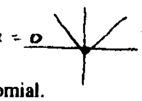
$x_0 = 1,\ y_0 = f(1) = 1$

$$\boxed{y-1 = -2(x-1)}$$

5) True/False. No justification needed.

(10 points total)

(a) If a function is continuous at a point, then it must also be differentiable there.

F e.g. $y=|x|$ at $x=0$  . (Reverse statement is true)

(b) The derivative of a polynomial is always a polynomial.

T (note: constant functions are polynomial

(c) For the number π, which is the area of the unit disk, the derivative of the function $f(x)=3\pi^4$ is $D_x f(x)=12\pi^3$

F $f(x)$ is constant function, so $f'(x)=0$

(d) If f(x) has derivative f'(2)=3, and value f(2)=5, then the derivative of the function $f(x)^4$ at x=2 is 1800.

F $D_x f(x)^4 = 4 f(x)^3 f'(x)$

@ $x=2$ get $4(5^3)(3) = 1500$

(e) The addition angle formula for sine is: $\sin(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$.

F $\sin(x+y) = \cos x \sin y + \sin x \cos y$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$.