#10
Assignment 3

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WeBWorK

Our records show problem 10 of set 3 has not been attempted.

(3 pts) **set3/math1210spring2003_korevaar_p3_9.pg**

The following problem illustrates several of the limit theorems on page 68 of the text, and shows how the limit concept is related to error analysis:

Suppose we have the estimates

$$|f(x)-2| < 0.03 \qquad |g(x)-5| < 0.03$$

Using only this information, make the best possible estimates for

(a) $|f(x)+g(x)-7| <$ .06

(b) $|f(x)g(x)-10| <$ .2109

(c) $|f(x)/g(x)-0.4| <$ .0845

work $|f(x)-2| < .03$

means $-.03 < f(x)-2 < .03$

+2: $\boxed{1.97 < f(x) < 2.03}$

similarly, $\boxed{4.97 < g(x) < 5.03}$

(a) add inequalities:

$1.97 + 4.97 < f(x)+g(x) < 2.03 + 5.03$

$6.94 < f(x)+g(x) < 7.06$

−7: $-.06 < f(x)+g(x)-7 < .06$

$\boxed{\text{so } |f(x)+g(x)-7| < .06}$

(b) since we are dealing with positive numbers, we see

$9.7909 \approx (1.97)(4.97) < f(x)g(x) < (2.03)(5.03) \cong 10.2109$

−10: $-.2101 < f(x)g(x) - 10 < .2109$

↑ larger error

so $\boxed{|f(x)g(x)-10| < .2109}$

(c) quotients of pos. numbers are largest when numerator is largest & denom is smallest, and analogous to minimize quotient.

Thus

$.39165 \approx \frac{1.97}{5.03} < \frac{f(x)}{g(x)} < \frac{2.03}{4.97} \approx .40845$

subtract $\frac{2}{5} = .4$:

$-.00835 < \frac{f(x)}{g(x)} - .4 < .00845$

↑ larger error

so $\boxed{\left|\frac{f(x)}{g(x)} - .4\right| < .00845}$