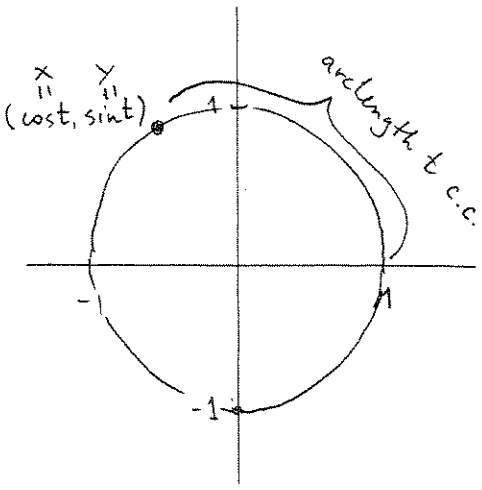
Math 1210-2
Wed Sept 5

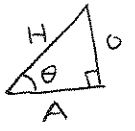
- Finish page 4 Tuesday: operations on functions

§0.6 Trigonometry review

The general definition of cost, sint:
Start at (1,0) on unit-radius circle centered at origin ("unit circle")
Traverse t units of arclength in the counter-clockwise direction.
(So if $t<0$ you go clockwise!).
The x-coord of the resulting point is cost. The y coord is sint



This generalizes SOHCAHTOA, which only worked for acute angles θ

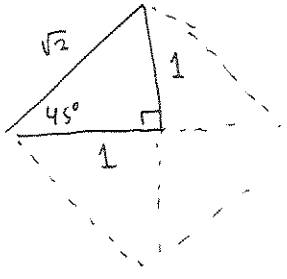


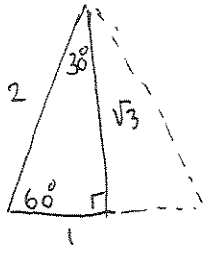
The arclength units t are called radians.
Since the perimeter of the radius 1 circle is 2π, we deduce 2π radians = 360 deg
i.e. $\boxed{\pi \text{ rad} = 180^\circ}$

Also, $\tan t := \frac{\sin t}{\cos t}$, $\sec t = \frac{1}{\cos t}$
$\cot t = \frac{\cos t}{\sin t}$, $\csc t = \frac{1}{\sin t}$

Exercise 1 Complete this table of sin, cos, tan for "basic" angles, which lets you deduce values for all angles shown on right

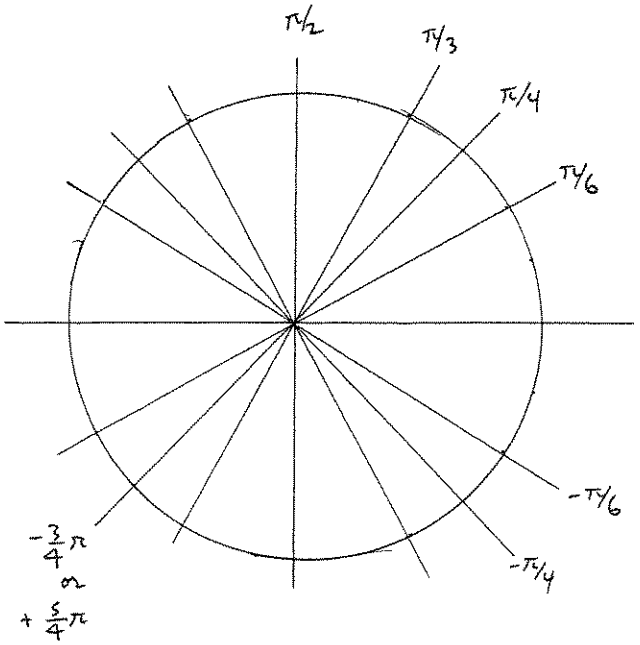
The table is on the next page, use your two favorite triangles for everything:



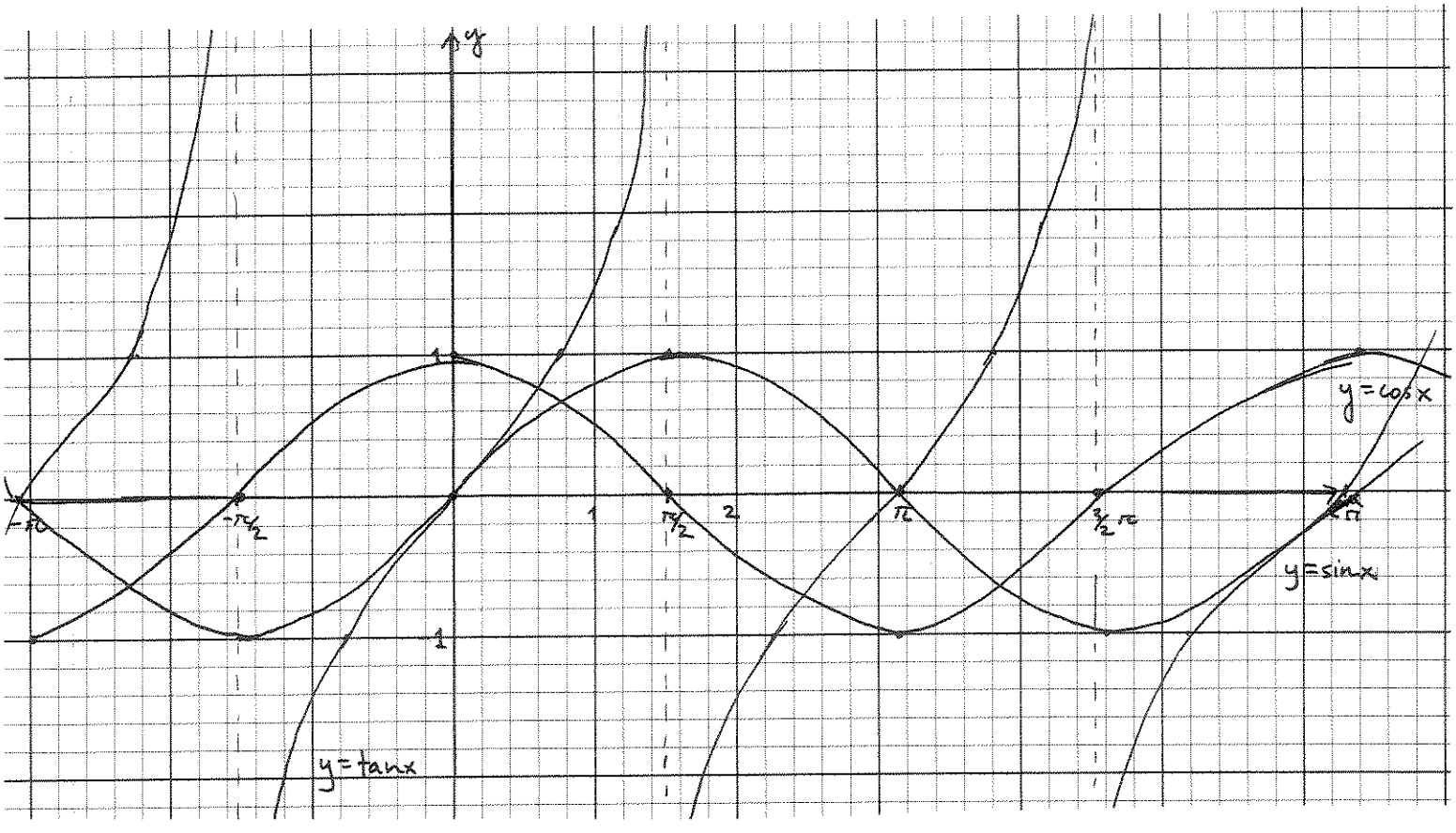


half an equilateral Δ

t rad	θ deg	cost	sint	tant	...
0	0	1	0	0	
$\pi/4$	45°				
$\pi/2$	90°				
$\pi/6$	30°				
$\pi/3$	60°				
$3\pi/4$	135°				
$\frac{5}{6}\pi$	150°				
⋮	⋮				
$-\frac{\pi}{4}$	-45°				
⋮	⋮				



Exercise 2 Describe how the graph $y = 2\cos(3x - \pi) + 1$ is related to $y = \cos x$, and add it to the sketches below!



trig identities pages 47-48.

There are 3 you should memorize, because all the others are consequences of these.

① $\sin^2 t + \cos^2 t = 1$ Pythagorean Theorem!

② $\sin(\alpha+\beta) = (\sin\alpha)(\cos\beta) + (\cos\alpha)(\sin\beta)$

③ $\cos(\alpha+\beta) = (\cos\alpha)(\cos\beta) - (\sin\alpha)(\sin\beta)$

} angle addition formula

Exercise

(a) Use ① to prove $\tan^2 t + 1 = \sec^2 t$

(b) Use ②, ③ to prove $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - (\tan\alpha)(\tan\beta)}$

Why ② ③ are true:

Exercise: Use geometry to fill in the missing coordinates, and deduce the identities for $\cos(\alpha+\beta)$ and $\sin(\alpha+\beta)$