Math 1210-2  
Wed Sept 5

* Finish page 4 Tuesday: operations on functions

4.6 Trigonometry review

The general definition of $\cos t, \sin t$:
Start at $(1,0)$ on unit-radius circle centered at origin ("unit circle")
Traverse $t$ units of archlength in the counter-clockwise direction.
(Does $t<0$ you go clockwise?).
The $x$-coord of the resulting point is $\cos t$. The $y$-coord is $\sin t$.

The archlength units $t$ are called radians.
Since the perimeter of the radius 1 circle is $2\pi$, we deduce $2\pi$ radians $= 360$ deg.
\[ \frac{7\pi}{2} \text{ rad} = 180^\circ \]

Also, $\tan t = \frac{\sin t}{\cos t}$, $\csc t = \frac{1}{\sin t}$,
$\cot t = \frac{\cos t}{\sin t}$, $\sec t = \frac{1}{\cos t}$.

Exercise 1
Complete this table of $\sin, \cos, \tan$ for "basic" angles, which lets you deduce values for all angles shown on right.
The table is on the next page, use your two favorite triangles for everything:

\[
\begin{align*}
\sqrt{2} & \\
45^\circ & 1 \\
\end{align*}
\]

\[
\begin{align*}
2 & \\
30^\circ & \sqrt{3} \\
60^\circ & 1 \\
\end{align*}
\]

Half an equivalent $\Delta$
<table>
<thead>
<tr>
<th>t (rad)</th>
<th>0</th>
<th>0</th>
<th>1</th>
<th>0</th>
<th>0</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>deg</td>
<td>0</td>
<td>0</td>
<td>45</td>
<td>90</td>
<td>30</td>
<td>60</td>
</tr>
<tr>
<td>$\frac{\pi}{4}$</td>
<td>$\frac{\pi}{2}$</td>
<td>$\frac{3\pi}{4}$</td>
<td>$\frac{5\pi}{6}$</td>
<td>$\frac{\pi}{2}$</td>
<td>$\frac{3\pi}{4}$</td>
<td>$\frac{5\pi}{6}$</td>
</tr>
<tr>
<td>$\frac{\pi}{6}$</td>
<td>$\frac{\pi}{6}$</td>
<td>$\frac{\pi}{6}$</td>
<td>$\frac{\pi}{6}$</td>
<td>$\frac{\pi}{6}$</td>
<td>$\frac{\pi}{6}$</td>
<td>$\frac{\pi}{6}$</td>
</tr>
</tbody>
</table>

**Exercise 2** Describe how the graph $y = 2\cos(3x - \pi) + 1$ is related to $y = \cos x$, and add it to the sketches below!
Trig identities pages 47-48.

There are 3 you should memorize, because all the others are consequences of these.

1. \( \sin^2 t + \cos^2 t = 1 \) \hspace{1cm} Pythagorean Theorem!
2. \( \sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \) \hspace{1cm} angle addition formula
3. \( \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \)

Exercise

(a) Use 1 to prove \( \tan^2 t + 1 = \sec^2 t \)
(b) Use 2, 3 to prove \( \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - (\tan \alpha)(\tan \beta)} \)

Why 2 3 are true:

Exercise: Use geometry to fill in the missing coordinates, and deduce the identities for \( \cos(\alpha + \beta) \) and \( \sin(\alpha + \beta) \)