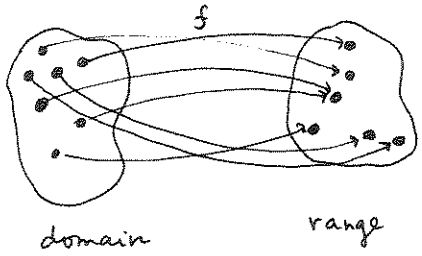
Math 1210-2
Tues Sept 4

§0.5-0.6
Functions, graphs of functions,
operations on functions

Definition A A <u>function</u> "f" is a rule of correspondence that associates with each object "x" in one set, called the <u>domain</u>, a single object f(x) from the second set. The set of all the objects f(x) so obtained is called the <u>range</u> of the function

<u>Example</u>: Let the domain be dairyman John's milk cows
Let f(x) be the favorite pasture location of cow x.
Then the range is the collection of all favorite locations

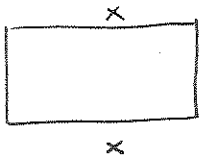
schematic picture for a function:



<u>Example</u> Let the domain be the students in 1210-2 (YOU!)
Let f(x) be student x's final course grade.
Then the range is the actual grades that get assigned, from A, A⁻¹, B⁺, ... D⁻, E, W

In Math 1210 our domain is (almost) always a subset of "x's" in the real numbers
and the range is (almost) always a subset of "y's" in the real numbers
(and in general, the domain elements are called <u>independent</u> variables,
and the range variables are called the <u>dependent</u> ones)
(since they are determined by the input x and the function f)

<u>Exercise 1</u> Find a formula for the function A(x), the area enclosed by a rectangular fence, if the total length of fence is 100 feet, and two opposite sides have length x.



A(x) =

domain is

range is

If a function formula is given without any domain specified, the domain is assumed to be the largest set possible, and called the <u>natural domain</u>

<u>Exercise 2</u> Find the natural domains of

(a) $g(x) = x(50-x)$

(b) $f(t) = \sqrt{9-t^2}$

(c) $h(s) = \sqrt{9-s^2} + \sqrt{s^2-16}$

A function is <u>even</u> if and only if $f(-x) = f(x)$
A function is <u>odd</u> iff $f(-x) = -f(x)$

<u>Exercise 3</u> : Are the following functions even, odd, or neither?
Why do you think the words <u>even</u> and <u>odd</u> were chosen for this concept?

(a) $f(x) = x^2 - 2$

(b) $g(x) = x^3 - x$

(c) $h(t) = 6t^8 + 4t^4 + 1$

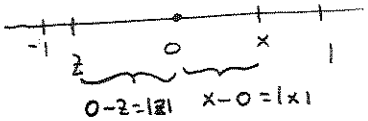
(d) $k(r) = r^5 + r^4$

<u>Exercise 4</u> An even function $f(x)$ has graph $y = f(x)$ which is symmetric with respect to _____

An odd function $g(x)$ has graph $y = g(x)$ which is symmetric with respect to _____

Two important functions to add to our "zoo"

the <u>absolute value function</u> $f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x \leq 0 \end{cases}$



geometrically $|x|$ is the distance from x to 0 on the numberline

$|3| = 3$
$|-2| = 2$
$|-.036| = .036$
etc.

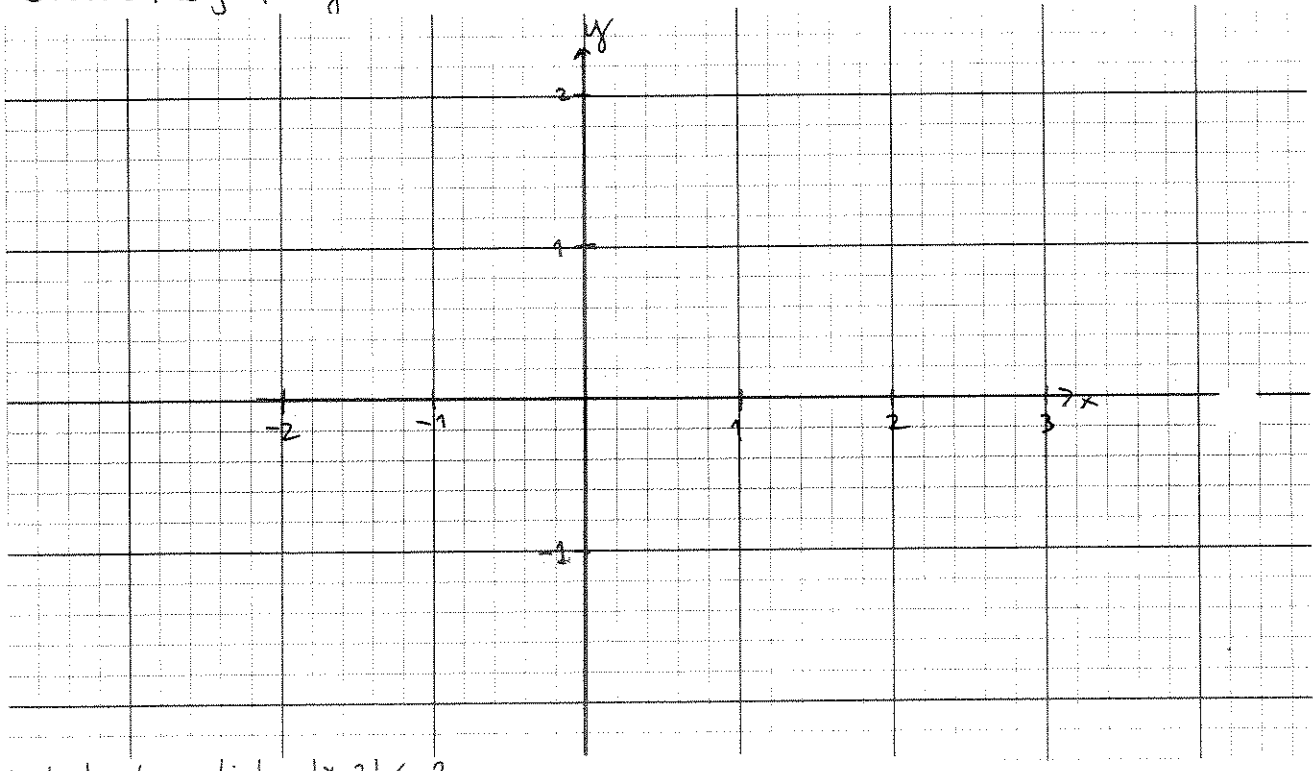
the <u>greatest integer function</u> $[\![x]\!]$ = the greatest integer $\leq x$

<u>Exercise 5</u>

(a) Sketch the graph $y = |x|$

(b) Sketch the graph $y = |x-3|$

(c) Sketch the graph $y = [\![x]\!]$



<u>Exercise 6</u>

(a) What x's satisfy $|x-3| < .2$

(b) If you were born at 1988.25 (about on April Fool's day), use the greatest integer function to express your age function, $a(t)$ your "age" at year t, i.e. how old you tell people you are

operations on real-valued functions

Let $f(x)$, $g(x)$ be functions.
Then the "new" functions

$$(f+g),\ fg,\ f/g,\ f\circ g$$

↑ sum ↑ product ↑ quotient ↖ composition

are defined by

$$(f+g)(x) = f(x) + g(x)$$
$$(fg)(x) = f(x)\,g(x)$$
$$(f/g)(x) = f(x)/g(x)$$
$$(f\circ g)(x) = f(g(x))$$

Exercise 7 Let $f(x) = x+2$, $g(x) = \frac{1}{x}$
What are

(a) $f+g$

(b) fg

(c) f/g

(d) $f\circ g$

(e) $g\circ f$

Exercise 8 Sketch the graphs (from Exercise 7) of f, g, $f+g$, $f\circ g$, $g\circ f$.
(i.e. the graphs $y=f(x)$, $y=g(x)$, ...)