Math 1210-2
Wednesday Sept 26
§ 2.6-2.7 Higher order derivatives
Implicit differentiation

Higher order derivatives

the derivative of the derivative of $f(x)$ is the second derivative of $f(x)$
the derivative of the second derivative is the third derivative, etc.

Notation!

We consider $y = f(x)$

Derivative	f' notation	y' notation	D notation	Leibniz notation	
first	f'	y'	$D_x y$	$\frac{dy}{dx}$	
second	f''	y''	$D_x^2 y$	$\frac{d^2y}{dx^2}$	("=" $\frac{d}{dx}\frac{dy}{dx}$)
third	f''' or $f^{(3)}$	y''' or $y^{(3)}$	$D_x^3 y$	d^3y/dx^3	("=" $\frac{d}{dx}\frac{d^2y}{dx^2}$)
fourth	$f^{(4)}$	$y^{(4)}$	$D_x^4 y$	d^4y/dx^4	
n^{th}	$f^{(n)}$	$y^{(n)}$	$D_x^n y$	d^ny/dx^n	

(we've already seen 2^{nd} derivatives when we did position-velocity-acceleration problems)

Exercise 1 If $y = \sin 2t$ Find

(a) y'

(b) $D_t^2 y$

(c) $\frac{d^3y}{dt^3}$

(d) $y^{(10)}(t)$

<u>Exercise 2</u> An object moves along a horizontal line in such a way that its position at time t is specified by

$$s = t^3 - 12t^2 + 36t - 30 \qquad \text{feet, at time } t \text{ seconds}$$

(a) When is the velocity zero?

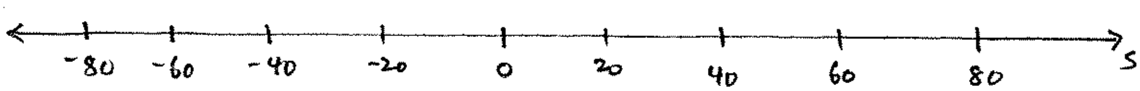
(b) When is the velocity positive?

(c) When is the object moving to the left, i.e. in the negative direction?

(d) when is the acceleration positive?

(e) Draw a picture to indicate the path of the object, onto the number line below.

t	s



Implicit differentiation

Often a function is implicitly defined by knowing an equation which the function satisfies. In this case (if the function is differentiable) one can deduce the derivative of the function by differentiating both sides of the equation (rather than by using an explicit formula for the function).
This is called implicit differentiation

Exercise 2 $f(x) = x^{p/q}$ where p, q are integers, $q \neq 0$, is a rational power function.

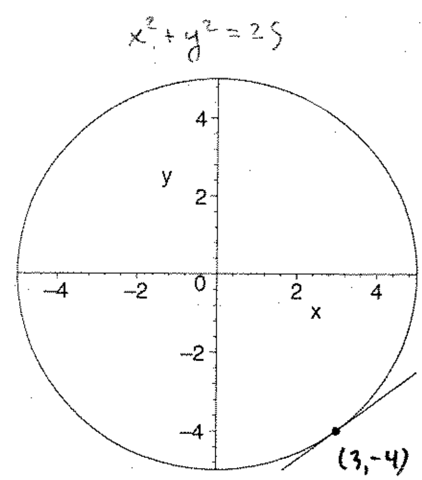
Find $f'(x)$ via implicit differentiation.
Note that $y = x^{p/q}$ satisfies the equation

$$y^q = x^p$$

Exercise 3 Consider the circle $x^2+y^2=25$. The upper half is the graph of the function $y=\sqrt{25-x^2}$, $-5 \le x \le 5$.
The lower half is the graph of $y=-\sqrt{25-x^2}$, $-5 \le x \le 5$

(a) Find the slope of the circle at $(3,-4)$ using the explicit formula $y=-\sqrt{25-x^2}$

(b) Repeat exercise (a) using implicit differentiation of the equation $x^2+y^2=25$ (much easier!!)



(c) Differentiate the result of (b) one more time, to find y'' at $(3,-4)$!