Math 1210-2
Tuesday Sept 29

§2.4–2.5 continued.

Exercise 1

$D_x\left(\frac{f(x)}{g(x)}\right) =$

$D_x\left(f(x)g(x)\right) =$

$D_x\left(f(g(x))\right) =$

Notations for the derivative

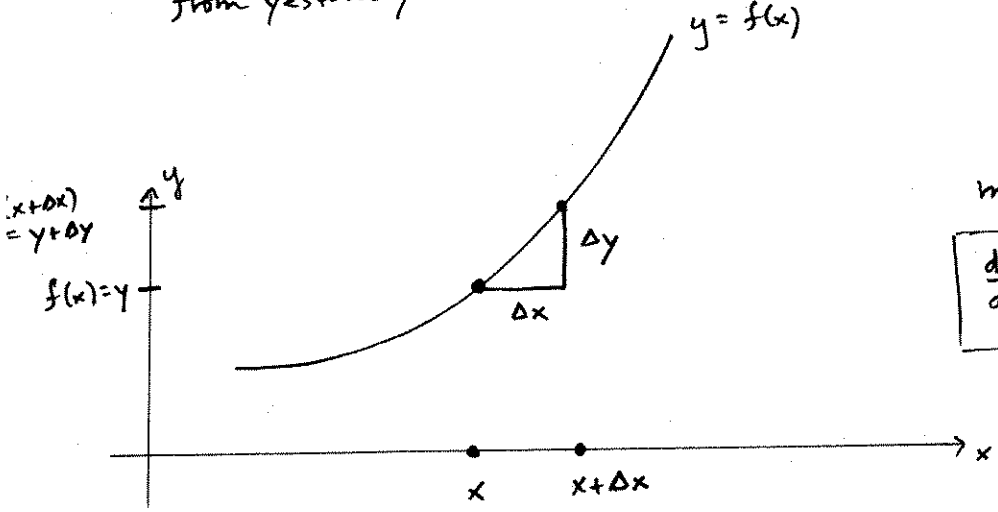
If $f(x)$ is a function of x, we've already seen

1. $f'(x)$
2. $D_x\, f(x)$

as expressions for the derivative of $f(x)$ with respect to the variable x.
A third notation (introduced by Leibniz, the co-inventor of Calculus, along with Newton) arises if we consider the graph $y = f(x)$. Then we may use

3. $\frac{dy}{dx}$ or $\frac{d}{dx} f(x)$

for the derivative. This is related to the $\frac{\Delta y}{\Delta x}$ secant-line slope descriptions from yesterday:



$m_{sec} = \frac{\Delta y}{\Delta x}$

$$\boxed{\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}}$$

$$\left(= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \right)$$

Δx and Δy could be positive or negative.
In this picture they're both positive

Often, when the chain rule is involved, the Leibniz notation appears and is useful. We'll see this later in §2.9 (differentials).
It's also the basis for the notation of antidifferentiation and (definite) integration.

Chain rule in Leibniz notation:

If $u = g(x)$
$y = f(u)$

then $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ $\xrightarrow{\text{same as}}$ $D_x f(g(x)) = f'(g(x))\, g'(x)$

In fact, this notation mirrors an (almost) proof of chain rule:

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \underbrace{\frac{\Delta y}{\Delta u}}_{\downarrow \; \frac{dy}{du}} \underbrace{\frac{\Delta u}{\Delta x}}_{\downarrow \; \frac{du}{dx}}$$

(since $\Delta u \to 0$ as $\Delta x \to 0$)

not quite a complete proof because doesn't account for the possibility that $\Delta u = 0$

Exercise 2 Practice switching notations, and computing

a) Find $G'(1)$ if $G(t) = (t^2+9)^3 (t^2-2)^4$:

b) $D_\theta \left[\cos^4(\sin\theta^2) \right]$

c) $\frac{dy}{dx}$ for $y = \left(\frac{\sin x}{\cos 2x} \right)^3$

<u>Exercise 3</u>: A Ferris wheel of radius 30 feet is rotating counterclockwise (from where you are watching). Its angular velocity is 2 radian/sec. How fast is a seat on the rim rising (in the vertical direction) when it is 15* feet above the center of the wheel?

a) Draw a picture!
b) Find $x(t)$ and $y(t)$, the location of the seat at time t
c) Answer the question.

Open floor: any webworks or HW problems to discuss?