Math 1210-2
Monday Sept 24
↳ 2.5 chain rule
↳ 2.4 trig fcn derivatives

xercise 1 Recall the differentiation rules we know so far

(i) $(f+g)' =$

(ii) $(cf)' =$ (c constant)

(iii) $(fg)' =$

(iv) $(f/g)' =$

(v) $(x^n)' =$ n any integer

(vi) $(\cos x)' =$

$(\sin x)' =$

$(\tan x)' =$

Exercise 2 A little fresh practice,

(i) $(\cot x)' =$

(ii) $(\sec x)' =$

(iii) $(\csc x)' =$

(iv) $(x^3 \sin x)' =$

f	f	f'
$\sin x$	✓	
$\cos x$	✓	
$\tan x$	$\frac{\sin x}{\cos x}$	
$\cot x$	$\frac{\cos x}{\sin x}$	
$\sec x$	$\frac{1}{\cos x}$	
$\csc x$	$\frac{1}{\sin x}$	

The chain rule

how to differentiate composite functions

warmup...

Exercise 3 If you can run twice as fast as your little brother and your dog can run 1.5 times as fast as you,
how much faster is your dog than your little brother?

Exercise 4 Use the graphs $u = g(x)$, $y = f(u)$ near $x=1$

to deduce the slope of $y = f(g(x))$ at $x = 1$. Sketch!
Assume all graphs are lines.

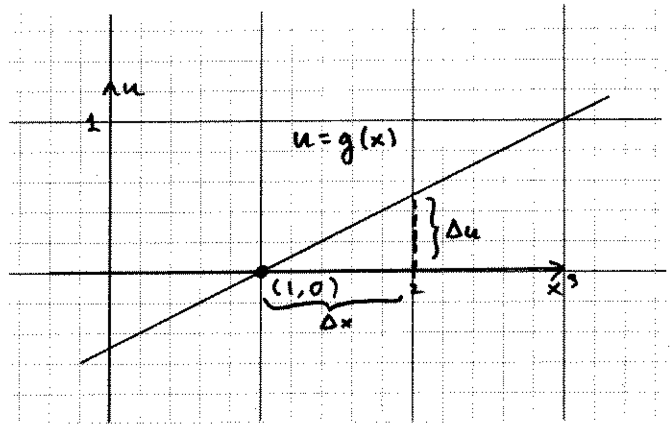
4a) $f(g(1)) =$

4b) $(f \circ g)'(1) =$

4c) Compute $f(g(x))$ explicitly (since graphs are exact lines), to check 4a,b

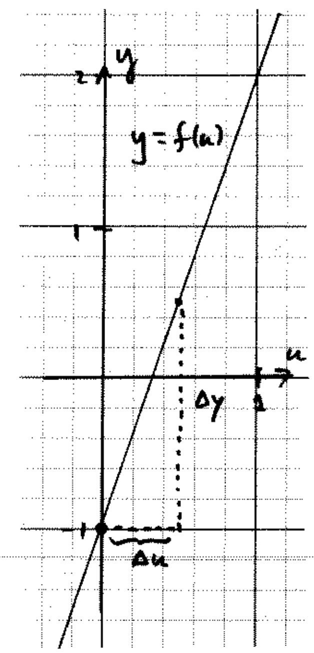
4d) Sketch $y = f(g(x))$

4e) Can you summarize the slope discussion in a simple way – like exercise 3?



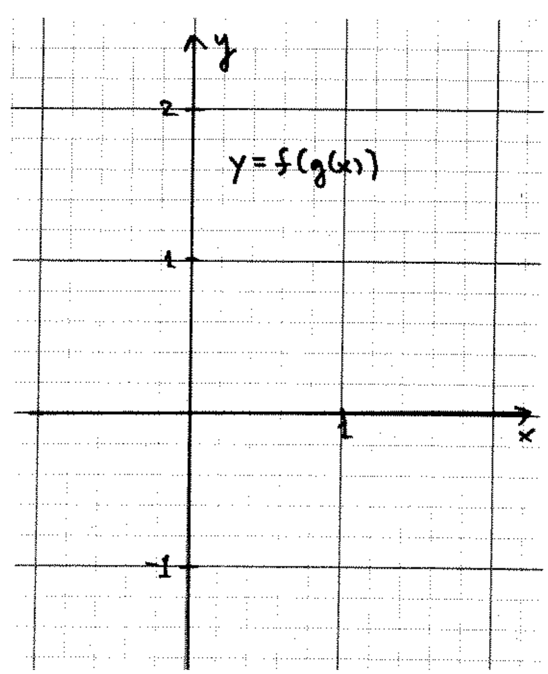
$g(1) = 0$
$g'(1) = ½$

$\frac{\Delta u}{\Delta x} = ½$



$f(0) = -1$
$f'(0) = 3$

$\frac{\Delta y}{\Delta u} = 3$



$\frac{\Delta y}{\Delta x} = \boxed{}$

Hopefully page 2 motivates...

<u>Chain Rule</u> For $u = g(x)$

$y = f(u) = f(g(x))$

Then

$$D_x\left(f(g(x)\right) = f'(g(x)) \cdot g'(x)$$

" The derivative with respect to x of $f(g(x))$ equals
the derivative of f, ~~at~~ evaluated at $g(x)$,
times $g'(x)$.

<u>Exercise 5</u> Write $F(x)$ as $f(g(x))$, and use the chain rule to differentiate $F(x)$.

5a) $F(x) = (x^3 + x)^2$

5b) $F(x) = \sin\left(\frac{x}{1+x^2}\right)$

5c) $F(x) = \left[\tan\left(x^3+x\right)\right]^2$! (this is why it's called the chain rule)

chain rule
Needs <u>lots</u> of practice