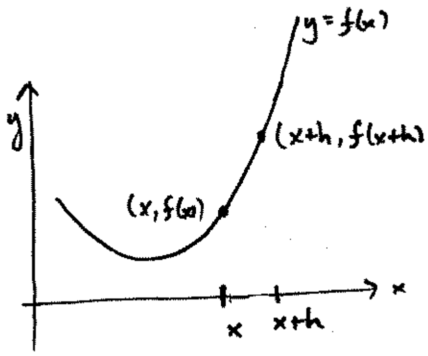
Math 1210-2
Wednesday Sept 19

§2.1 Derivatives and rates of change.



We know the definition for derivative!

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

We call $\frac{f(x+h)-f(x)}{h}$ the average slope between x and x+h
or
the average rate of change of f(x), between x and x+h

the derivative $f'(x)$ is the limit of these average slopes (or rates of change), and is sometimes called the instantaneous rate of change of f with respect to x

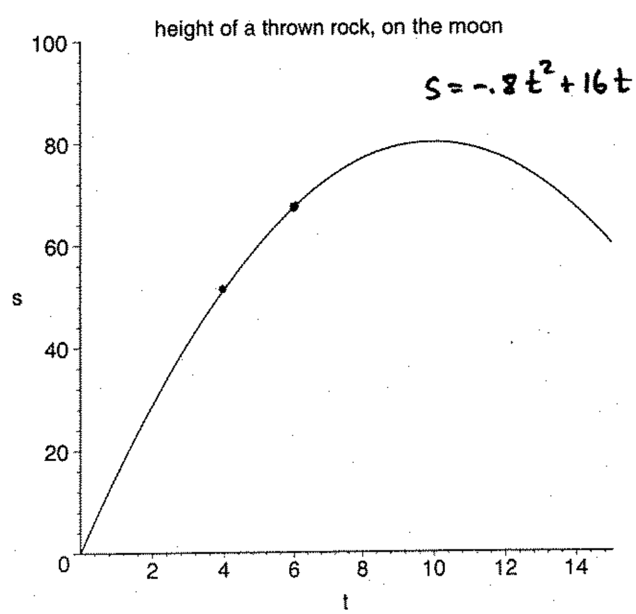
We know this sort of specific example!

The acceleration of gravity on the moon is about 1.6 meters per second per second, so if a rock is thrown vertically (are you tired of only studying dropped objects, or vertically thrown ones?), with initial velocity of 16 meters per second, and initial height, the height function at time t seconds later will be given by

$$s(t) = -0.8\, t^2 + 16\, t.$$

Here is a graph of the height function, for the first 15 seconds:

Exercise 1 (a) Find the average rate of change of height ("average velocity") between t=4 and t=6 seconds. Include units!



(b) What is the velocity at t=4 sec,? i.e. the instantaneous rate of change of position?

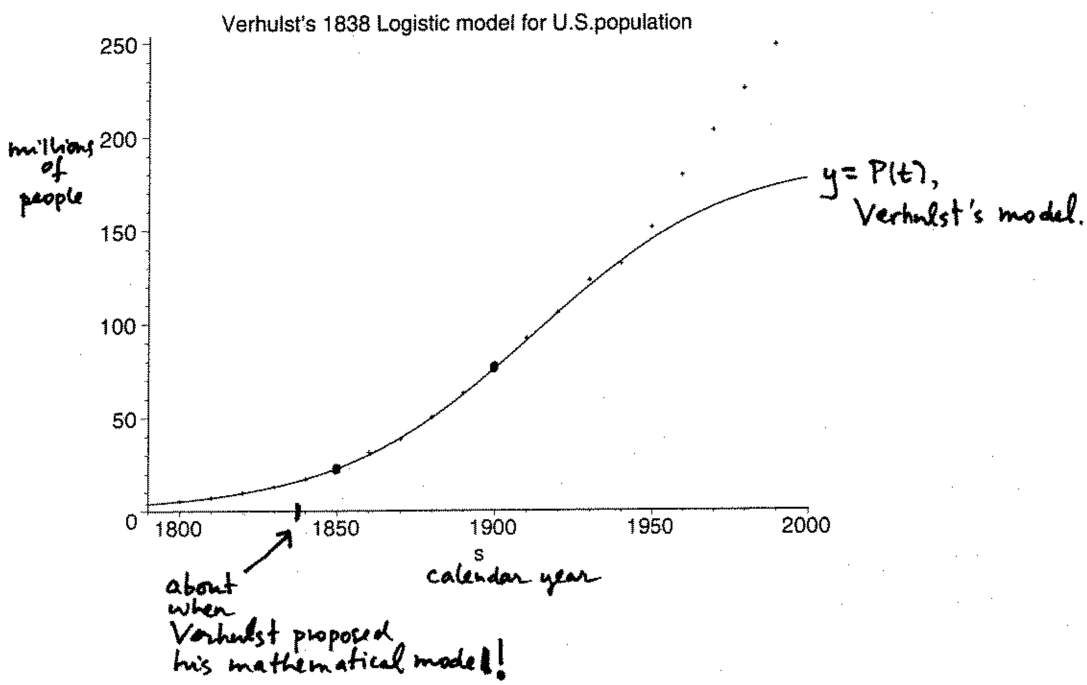
Derivatives = rates of change, cont'd:

In the 1830's a Belgian Demographer Pierre Verhulst proposed that many populations follow what is now called the logistic model of population growth:

$$P'(t) = rP(t)\left(1 - \frac{P(t)}{k}\right).$$

He found good fits for the rate parameter r, and the "carrying capacity" k, based on ~~population~~ U.S. population data from several preceeding decades, and then found a solution function to solve this equation. That led to predictions for future U.S. population numbers. This example illustrates the effectiveness and limitations of mathematical modeling, especially for population problems!

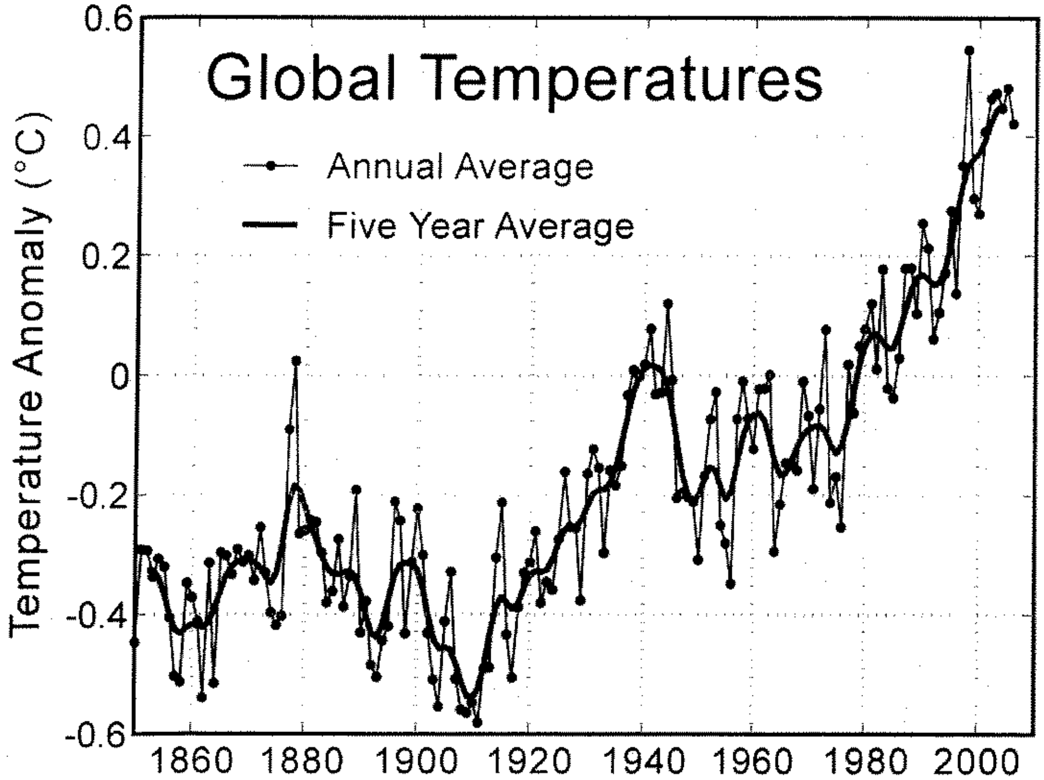
The vertical axis units are millions of people. The horizontal axis units are calendar years.
points represent actual data. Smooth graph is Verhulst's prediction.



Exercise 2a) Use the graph and data above to estimate the average rate of change of population ("population growth rate") between 1850 and 1900. Include units!

2b) Estimate the exact population growth rate in 1900.

2c) What happened around 1950 that messed up Verhulst's model, which had been amazingly accurate until then?



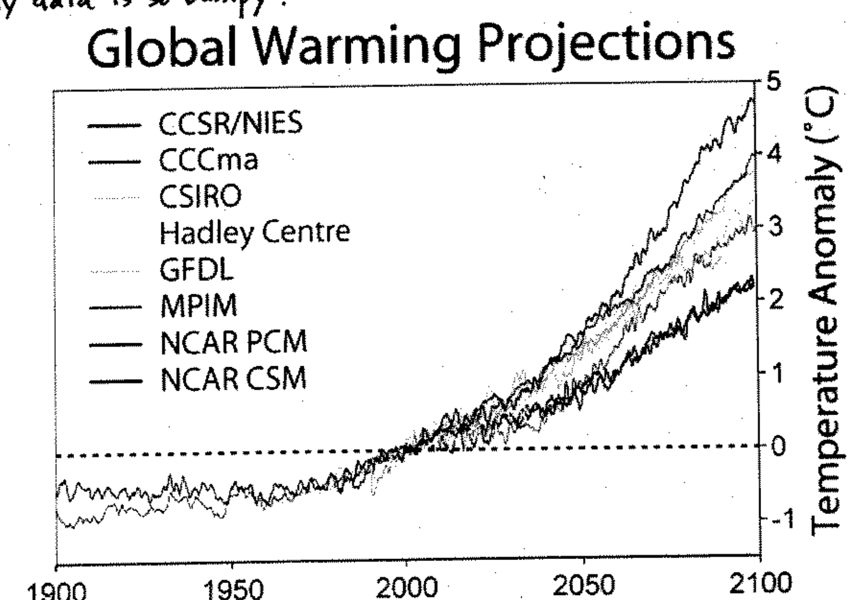
http://en.wikipedia.org/wiki/Global_warming

" Global mean surface temperature anomaly 1850-2006 relative to 1961-1990 "

Exercise 3   According to this graph, estimate the average rate at which the Earth's mean surface temperature increased, between 1980 and 2000?

Exercise 4 : Why do you think the yearly data is so bumpy?

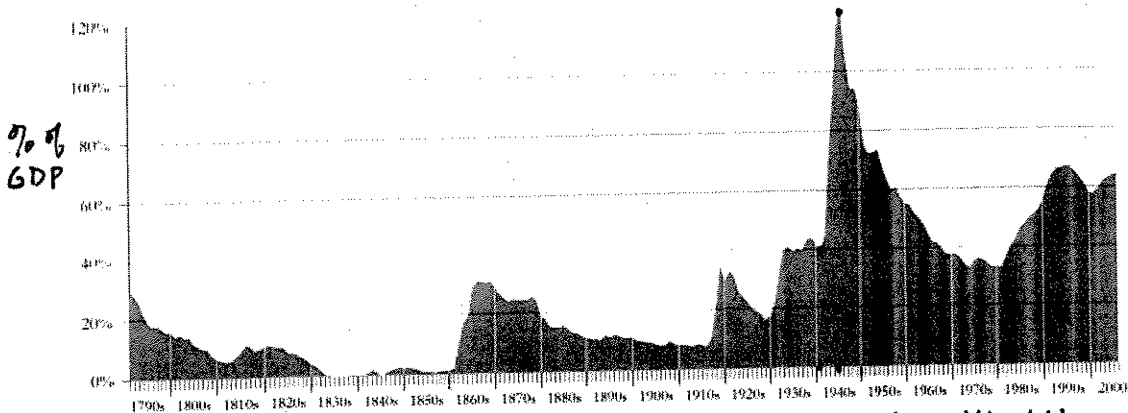
This one is tough to model!



(same wikipedia page.)

This example is probably impossible to model precisely, because it is almost completely dependent on human behavior! But, just understanding the language, and what it means mathematically, is necessary to avoid being misled.

Federal Public Debt as Percentage of GDP, 1791–2006



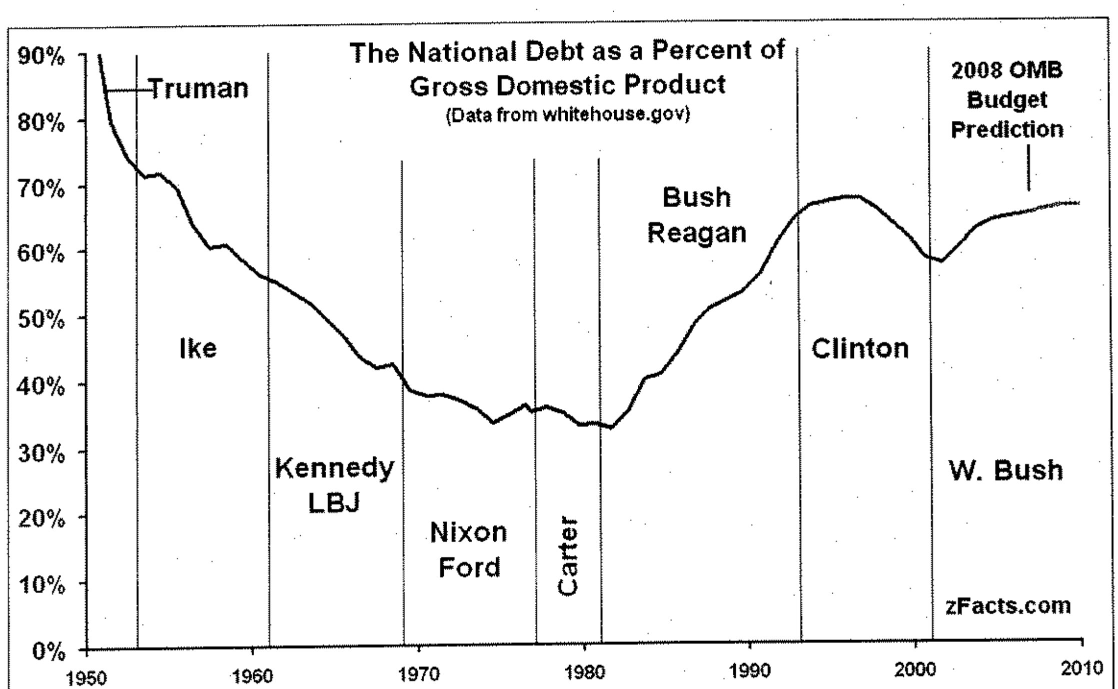
http://en.wikipedia.org/wiki/United_States_public_debt

Exercise 5 At what average rate did the U.S. national debt rise, as a percent of gross domestic product (= annual total ~~of~~ value of all goods & services produced), between 1941 & 1945? What event spurred this?

Exercise 6 The current national debt is just over \$9 trillion (9×10^{12} dollars), about \$30,000 per person.

The "deficit" decreased from \$412 billion in 2004 to \$248 billion in 2006?

What happened to the debt?



http://zfacts.com/p/318.html