Math 1210-2
Monday Sept 17

Continue limits!
§1.4 trig limits

- Review page 3 Wed 9/12
  - D. squeeze theorem
  - E. limit statements

Do Exercise 5.

Exercise 1: Use $\lim_{t\to 0} \sin t = 0$ $(= \sin 0)$

$\lim_{t\to 0} \cos t = 1$ $(= \cos 0)$

and limit theorems to check

$$\lim_{x\to c} \cos x = \cos c$$
$$\lim_{x\to c} \sin x = \sin c$$
$$\lim_{x\to c} \tan x = \tan c \quad (\text{as long as } \cos c \neq 0)$$

you might need trig identities too!

Definition $f(x)$ is continuous at $x=c$ if and only if $\lim_{x\to c} f(x) = f(c)$

So far, we have shown polynomials are continuous at every $x=c$, also $\sin x$, $\cos x$, also rational functions at pts where the denom $\neq 0$.

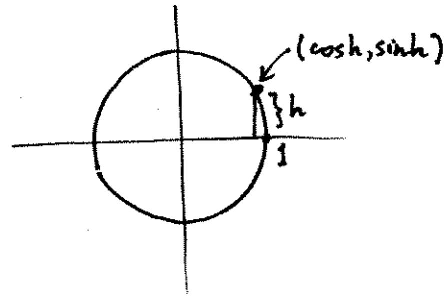
(If $f(x)$ is continuous at $x=c$ you can compute $\lim_{x\to c} f(x)$ by "plugging in" $f(c)$.)

Important trig limits:

(a) $\lim_{h\to 0} \frac{\sin h}{h} = 1$

(b) $\lim_{h\to 0} \frac{1-\cos h}{h^2} = \frac{1}{2}$

(c) $\lim_{h\to 0} \frac{1-\cos h}{h} = 0$

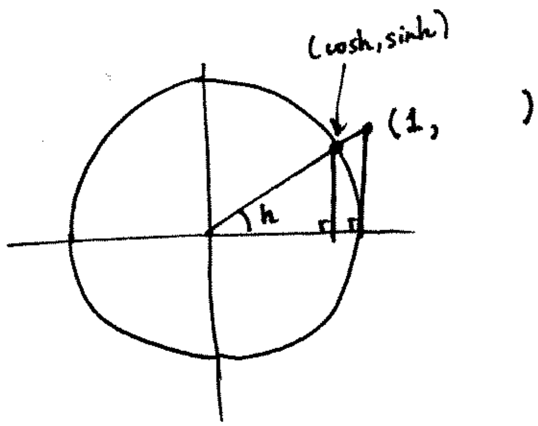


Exercise 2: Notice than $\frac{\sin h}{h}$ is even, since $\frac{\sin(-h)}{-h} = \frac{-\sin h}{-h} = \frac{\sin h}{h}$.

Therefore, to see (a), we need only check

$$\lim_{h\to 0^+} \frac{\sin h}{h} = 1.$$

Use the following diagrams and area formulas, and the squeeze theorem to prove this fact.



Exercise 3: Use the double angle formula for cos, in the form $\cos(2\frac{h}{2}) = 1 - 2\sin^2 h/2$ to prove (b)

Exercise 4 Deduce (c) from (b)

Limit (a) leads to lots of "fun" limits, see e.g. §1.4 problems.

Exercise 5 (#8 §1.4)

$$\lim_{\theta \to 0} \frac{\tan 5\theta}{\sin 2\theta}$$

(#11 §1.4)

$$\lim_{t \to 0} \frac{\tan^2 3t}{2t}$$

Exercise 6 :

$$(\cos x)' = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$(\sin x)' = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

Compute these limits!