Math 12.10-2  
Monday Sept 17  
Continue limits!  
9.1.4 trig limits  
* Review page 3 Wed 9/2  
  D. squeeze theorem  
  E. limit statements  
  Do Exercise 5.  

Exercise 1: Use \( \lim_{t \to 0} \sin t = 0 \) and \( \lim_{t \to 0} \cos t = 1 \) and limit theorems to check  
\[ \lim_{x \to c} \cos x = \cos c \]  
\[ \lim_{x \to c} \sin x = \sin c \]  
\[ \lim_{x \to c} \tan x = \tan c \quad \text{(as long as } \cos c \neq 0) \]  
you might need trig identities too!  

Definition \( f(x) \) is \underline{continuous} at \( x = c \) if and only if \( \lim_{x \to c} f(x) = f(c) \)  
So far, we have  
shown polynomials are continuous at every \( x = c \),  
also \( \sin x, \cos x, \) also rational  
functions at pts where the denomon\( \neq 0 \).  

(If \( f(x) \) is continuous at \( x = c \) you can compute \( \lim_{x \to c} f(x) \) by "plugging in" \( f(c) \).)
Important trig limits:
(a) \( \lim_{h \to 0} \frac{\sinh h}{h} = 1 \)
(b) \( \lim_{h \to 0} \frac{1 - \cosh h}{h^2} = \frac{1}{2} \)
(c) \( \lim_{h \to 0} \frac{1 - \cosh h}{h} = 0 \)

Exercise 2: Notice that \( \frac{\sinh h}{h} \) is even, since \( \frac{\sin(-h)}{-h} = \frac{-\sinh h}{-h} = \frac{\sinh h}{h} \).

Therefore, to see (a), we need only check \( \lim_{h \to 0^+} \frac{\sinh h}{h} = 1 \).

Use the following diagrams and area formulas, and the squeeze theorem to prove this fact.

Exercise 3: Use the double angle formula for \( \cos \), in the form \( \cos (2 \frac{h}{2}) = 1 - 2 \sin^2 \frac{h}{2} \) to prove (b).

Exercise 4: Deduce (c) from (b)
Limit (a) leads to lots of fun limits, see e.g. §1.4 problems.

Exercise 5  \((\#8 \text{ §1.4})\)
\[
\lim_{\theta \to 0} \frac{\tan 5\theta}{\sin 2\theta}
\]
\((\#11 \text{ §1.4})\)
\[
\lim_{t \to 0} \frac{\tan^2 3t}{2t}
\]

Exercise 6:
\[
(\cos x)' = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}
\]
\[
(\sin x)' = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}
\]

Compute these limits!