Math 1210-2
Wed. Sept 12

Test Friday!
Practice exam on-line
(solutions will be posted late today)
Review sheet in today's notes
Review session Thurs 9:40-10:30 here,
mostly to go over practice exam.

**Exercise 1** 
What is the $\varepsilon$-$\delta$ definition of \( \lim_{x \to c} f(x) = L \)?

**Exercise 2** 
Review the main limit theorem "A" from Tuesday notes,
and use it to do step by step justification, in finding

\[ 2a) \lim_{x \to -1} 3x^2 + x \]

\[ 2b) \lim_{x \to 3} \frac{\sqrt{x^2+9}}{x} \]

\[ 2c) \lim_{x \to 2} \frac{3}{x-8} \frac{x-8}{x^2-4} \text{ Careful!} \]

*Use Theorem C*

If \( f(x) = g(x) \) for all \( x \) in an open interval containing \( c \), except possibly at \( c \) itself, and
if \( \lim_{x \to c} g(x) \) exists, then \( \lim_{x \to c} f(x) \) exists too, with
\( \lim_{x \to c} f(x) = \lim_{x \to c} g(x) \) the same limit.
recall, a polynomial function \( f \) has the form 
\[
    f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0
\]

a rational function \( f \) is a quotient of two polynomials,
\[
    f(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \cdots + b_1 x + b_0}
\]

**Theorem B** Substitution Theorem for rational functions

If \( f \) is a polynomial or rational function, then

\[
    \lim_{x \to c} f(x) = f(c),
\]

provided \( f(c) \) is defined. For a rational function, this means the denominator is non-zero for \( x = c \).

**Exercise 3** \[
    \lim_{x \to -2} \frac{x^3 + 8x^2 - 7}{x^2 + 4} =
\]

**Exercise 4** \[
    \lim_{x \to 2} \frac{x^2 + 3x - 10}{x^2 + x - 6} =
\]

**Theorem D** Squeeze Theorem: Let \( f(x), g(x), h(x) \) satisfy \( f(x) \leq g(x) \leq h(x) \) for all \( x \) is an open interval containing \( c \), except possibly at \( x = c \), and if \( \lim_{x \to c} f(x) = \lim_{x \to c} h(x) = L \), then also \( \lim_{x \to c} g(x) = L \).

**Proof:** Let \( \varepsilon > 0 \).

Pick \( \delta_1 > 0 \) so that

\[
    0 < |x - c| < \delta_1 \Rightarrow L - \delta < f(x) < L + \delta
\]

Pick \( \delta_2 > 0 \) so that

\[
    0 < |x - c| < \delta_2 \Rightarrow L - \delta < h(x) < L + \delta
\]

Let \( \delta = \text{minimum of } \delta_1, \delta_2 \). Then

\[
    0 < |x - c| < \delta \Rightarrow L - \delta < g(x) < L + \delta
\]

\[
    L - \varepsilon < g(x) < L + \varepsilon
\]
Theorem E  The following limit statements are equivalent:

(1) \( \lim_{x \to c} f(x) = L \)

(2) \( \lim_{x \to c} |f(x) - L| = 0 \)

(3) \( \lim_{h \to 0} |f(c+h) - L| = 0 \)

Exercise 5  Use the squeeze theorem and Theorem E to prove

(1) \( \lim_{t \to 0} \sin t = 0 \)

(2) \( \lim_{t \to 0} \cos t = 1 \).

Begin with the geometric fact (from the Pythagorean Theorem) that,

\[ 0 \leq (1 - \cos^2 t) + \sin^2 t \leq t^2 \]
Exam 1 Review Sheet: to cover P.1-P.5

0.4-0.7
0.1-0.3
1.1-1.3

Practice problem sources:
Webwork 1-3
recommend problems from sections
practice midterm (posted)
weekly quizzes & good sources!

Topics:

1.1-1.5 lines
slopes
slope-intercept form, pt-slope form, general eqn.
L lines, // lines

Slopes of graphs y = f(x)
limit definition of slope function, i.e. of derivative
(also, for position on s(t), average velocities and
instantaneous vel.)
derivatives of polynomials
antidifferentiation
FTC for area:
working backwards from area to position

0.4-0.7 graphs of equations
how to translate, scale, reflect graphs by modifying
equations

functions

domain
range
graphs of functions, and
trig functions, especially sin, cos, translations & scalings
trig identities — the 3 you need.

1.1-1.3 limits
computing easy or tricky limits — by substitution, before or after
cancelling common terms if necessary.

The e-δ game, error estimation
limit theorems