Math 1210-2
Wed Sept 12

Test Friday!
Practice exam → on-line
(solutions will be posted late today)
Review sheet in today's notes
Review session Thurs 9:40-10:30 here,
mostly to go over practice exam.

Exercise 1 : What is the $\varepsilon-\delta$ definition of $\lim_{x \to c} f(x) = L$?

Exercise 2 : Review the main limit theorem "A" from Tuesday notes, and use it to do step by step justification, in finding

2a) $\lim_{x \to -1} 3x^2 + x$

2b) $\lim_{x \to 3} \frac{\sqrt{x^2+9}}{x}$

2c) $\lim_{x \to 2} \frac{x^3 - 8}{x^2 - 4}$ Careful!

Use Theorem C If $f(x) = g(x)$ for all x in an open interval containing c, except possibly at c itself, ~~the~~ and if $\lim_{x \to c} g(x)$ exists, then $\lim_{x \to c} f(x)$ exists too, with the same limit.

recall, a polynomial function f has the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$

a rational function f is a quotient of two polynomials,

$$f(x) = \frac{a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \dots + b_1 x + b_0}$$

Theorem B Substitution Theorem for rational functions

If f is a polynomial or rational function, then

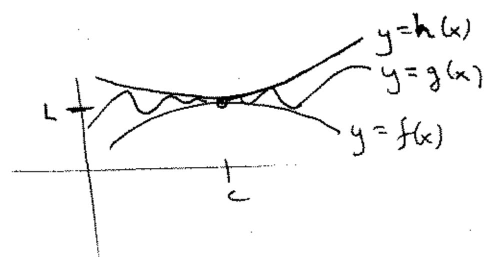
$$\lim_{x \to c} f(x) = f(c),$$

provided $f(c)$ is defined. For a rational function this means the denominator is non-zero for $x=c$.

Exercise 3 $\displaystyle\lim_{x \to -2} \frac{x^3 + 8x^2 - 7}{x^2 + 4} =$

Exercise 4 $\displaystyle\lim_{x \to 2} \frac{x^2 + 3x - 10}{x^2 + x - 6} =$

Theorem D Squeeze Theorem: Let $f(x), g(x), h(x)$ satisfy $f(x) \le g(x) \le h(x)$ for all x is an open interval containing c, except possibly at $x=c$, and if $\displaystyle\lim_{x\to c} f(x) = \lim_{x \to c} h(x) = L$, then also $\displaystyle\lim_{x \to c} g(x) = L$.



proof: Let $\varepsilon > 0$.

Pick $\delta_1 > 0$ so that

$0 < |x-c| < \delta_1 \Rightarrow L - \varepsilon < f(x) < L + \varepsilon$

Pick $\delta_2 > 0$ so that

$0 < |x-c| < \delta_2 \Rightarrow L - \varepsilon < h(x) < L + \varepsilon$

Let δ = minimum of δ_1, δ_2. Then

$0 < |x-c| < \delta \Rightarrow L - \varepsilon < f(x) < g(x) < h(x) < L + \varepsilon$

$\Rightarrow L - \varepsilon < g(x) < L + \varepsilon$ ∎

Theorem E The following limit statements are equivalent:

check:

(1) $\lim_{x \to c} f(x) = L$

(2) $\lim_{x \to c} |f(x) - L| = 0$

(3) $\lim_{h \to 0} |f(c+h) - L| = 0$

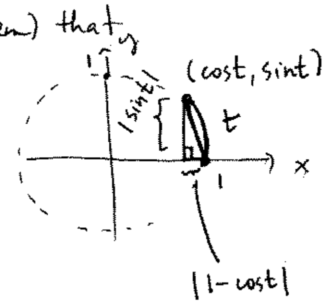
Exercise 5 Use the squeeze theorem and Theorem E and maybe Theorem A to prove

(1) $\lim_{t \to 0} \sin t = 0$

(2) $\lim_{t \to 0} \cos t = 1.$

Begin with the geometric fact (from the Pythagorean Theorem) that

$$0 \leq (1-\cos t)^2 + \sin^2 t \leq t^2$$



Exam 1 Review Sheet : to cover $\boxed{\begin{matrix}P.1-P.5\\ 0.4-0.7\\ 1.1-1.3\end{matrix}}$

practice problem sources : Webworks 1–3
recommend problems from sections
practice midterm (posted)
weekly quizzes ← good sources!

topics

P.1–P.5 lines
- slope
- slope-intercept form, pt-slope form, general eqtn
- ⊥ lines, // lines

slopes of graphs $y = f(x)$
- limit definition of slope function, i.e. of derivative
  - (also, for position fcn $s(t)$, average velocities and instantaneous vel.) also, acceleration
- derivatives of polynomials

antidifferentiation
- FTC for area
- working backwards from accel or vel. to position

0.4–0.7 graphs of equations
- how to translate, scale, reflect graphs by modifying equations

functions
- domain
- range
- graphs of functions, and
- trig functions, especially $\sin x$, $\cos x$, translations & scalings
  - trig identities → the 3 you need.

1.1–1.3 limits
- computing easy or tricky limits → by substitution, before or after cancelling common terms if necessary.
- precise ε-δ limit def.
  - the ε-δ game, error estimation
- limit theorems