Name: SOLUTIONS

UID

Math 1210-2

Quiz 8

November 9, 2007

Show all work for complete credit!

1a) What is the statement of the Mean Value Theorem, for a function f(x) which is continuous on the closed interval [a, b] and differentiable on the open interval (a, b) ?

(1 point)

There exists at least one c, $a < c < b$, so that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

1b) Verify the Mean Value Theorem, for $f(x) = \frac{1}{x}$, $1 \le x \le 3$.

(3 points)

$$f'(x) = -\frac{1}{x^2}$$

$$\frac{f(b)-f(a)}{b-a} = \frac{f(3)-f(1)}{3-1} = \frac{\frac{1}{3}-1}{2} = \frac{-2/3}{2} = -1/3$$

solve $-\frac{1}{3} = -\frac{1}{x^2} \Rightarrow x^2 = 3 \Rightarrow \boxed{x = \sqrt{3} = c}$

$(1 < c < 3)$

($x = -\sqrt{3}$ not in our interval)

2) Compute the following antiderivatives:

2a)

$$\int x^2 + 3\sin(2x) + \frac{1}{x^3}\,dx$$

x^{-3}

(3 points)

$$F(x) = \frac{x^3}{3} + \frac{3}{2}(-\cos(2x)) + \frac{x^{-2}}{-2} + C$$

Check: $F'(x) = \frac{1}{\cancel{3}}\cdot\cancel{3}x^2 + \frac{3}{\cancel{2}}(\cancel{-}\sin(2x)\cancel{2}) + (\cancel{-\tfrac{1}{2}})(\cancel{-2})x^{-3}$ ✓

$+$

2b)

$$\int \frac{7t^2}{\sqrt{10+15t^3}}\,dt$$

(3 points)

$u = 10 + 15t^3$

$du = 45t^2\,dt$

$\frac{du}{45} = t^2\,dt$

$$= \int \frac{7\left(\frac{du}{45}\right)}{\sqrt{u}} = \frac{7}{45}\int u^{-1/2}\,du = \frac{7}{45}\frac{u^{1/2}}{1/2} + C$$

$$= \frac{14}{45}u^{1/2} + C = \frac{14}{45}\sqrt{10+15t^3} + C$$

(Check: $F'(t) = \frac{14}{\cancel{45}}\frac{1}{2}(10+15t^3)^{-1/2}\cancel{15}\cdot\cancel{3}t^2 = \frac{7t^2}{\sqrt{10+15t^3}}$ ✓)