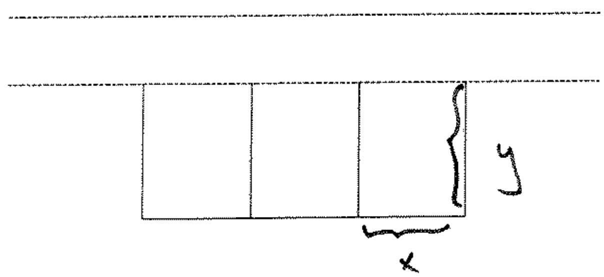
Name: Solutions

UID:

Math 1210-2
Quiz #6
October 26, 2007

Show all work for complete credit!

1) Farmer Joe wishes to test three new strains of table corn, as well as advertise his farming abilities. He will fence off three adjacent and congruent rectangular plots, one for each variety of corn, using the road as the "northern" (fenceless) border for each plot, as indicated in the diagram below:



Farmer Joe happens to have 600 linear feet of fencing available to construct his plots. What dimensions should he choose for each rectangular plot in order to maximize its area?

1a) Find the answer to the question above using Calculus. (7 points)

Maximize $A = xy$

subject to $600 = 3x + 4y$

$$y = \frac{600-3x}{4} = 150 - \frac{3}{4}x$$

so $A(x) = x\left(150 - \frac{3}{4}x\right) = 150x - \frac{3}{4}x^2$

$$A'(x) = 150 - \frac{3}{2}x = -\frac{3}{2}\left(x - 150\left(\frac{2}{3}\right)\right) = -\frac{3}{2}(x-100).$$

$A'(x) = 0$ at $x = 100 \Rightarrow y = 150 - 75 = 75$

Choose $x = 100$, $y = 75$

1b) Explain logically why your answer must be correct, using concepts we've been discussing. (Use the back of the page if necessary). (3 points)

Any of the following reason's are acceptable

① sign A': $+ + + + 0 - - -$ (at 100), so $A(x)$ is inc $x \le 100$, dec $x \geq 100$. so $A(100)$ is global max

② domain $0 \le x \le 200$
$A(0) = A(200) = 0$. A is diff'ble
so max value occurs at critical point: can't be endpoint, can't be singular point, must be stationary point. $x = 100$ is the only one!

③ $A''(x) = -3/2$, so $y = A(x)$ is CD for all x, so there can be only one stationary point, and it must be location of local max.

④ $y = A(x)$ is a [illegible] parabola. It's vertex height is the global max value.